An aerial photograph of a winding asphalt road through a lush green mountain valley. The road curves and loops through the terrain, which is covered in dense vegetation and some rocky patches. The background shows more of the valley and distant hills under a clear sky.

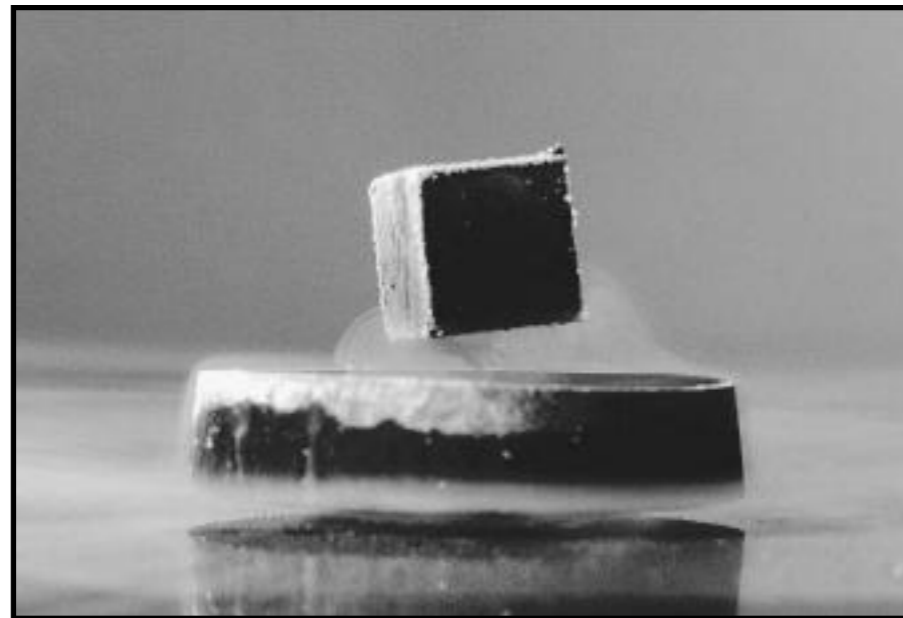
The long road to an ideal realization of the kagome lattice antiferromagnet: a few perturbations met along the way...

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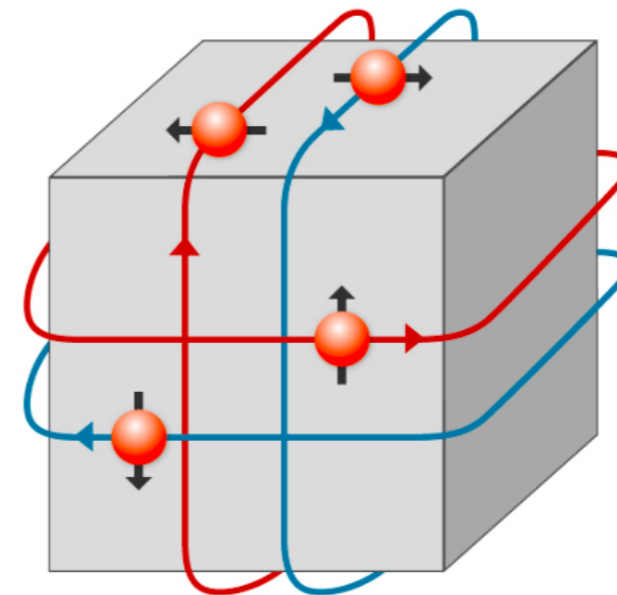
Gøran Nilsen  
ISIS Facility, Rutherford Appleton Laboratory, UK

Materials that show macroscopic quantum behaviour because of electron correlations and/or geometry/dimensionality

*e.g.* high- $T_c$  superconductors



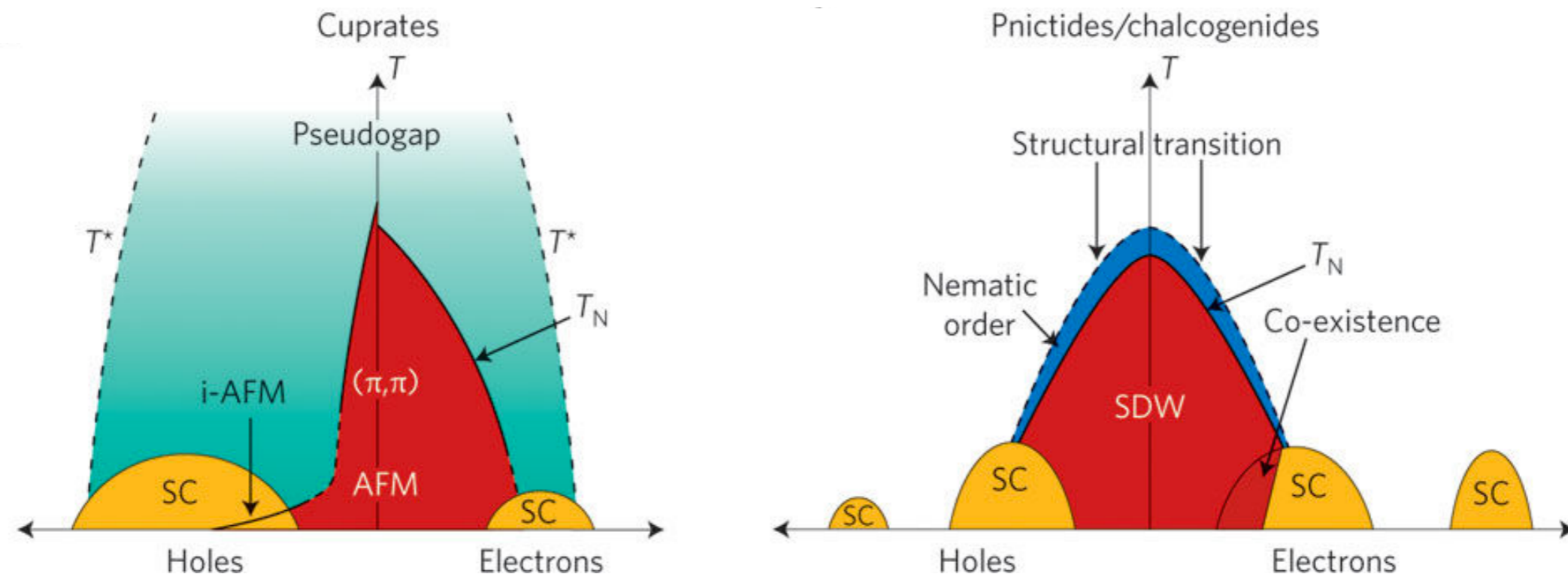
*e.g.* topological insulators



Keimer et. al. Nature Phys. **13** 1046; Tokura et. al. Nature Phys. **13** 1056

Enormous promise for future energy transport, sensing, computing etc.

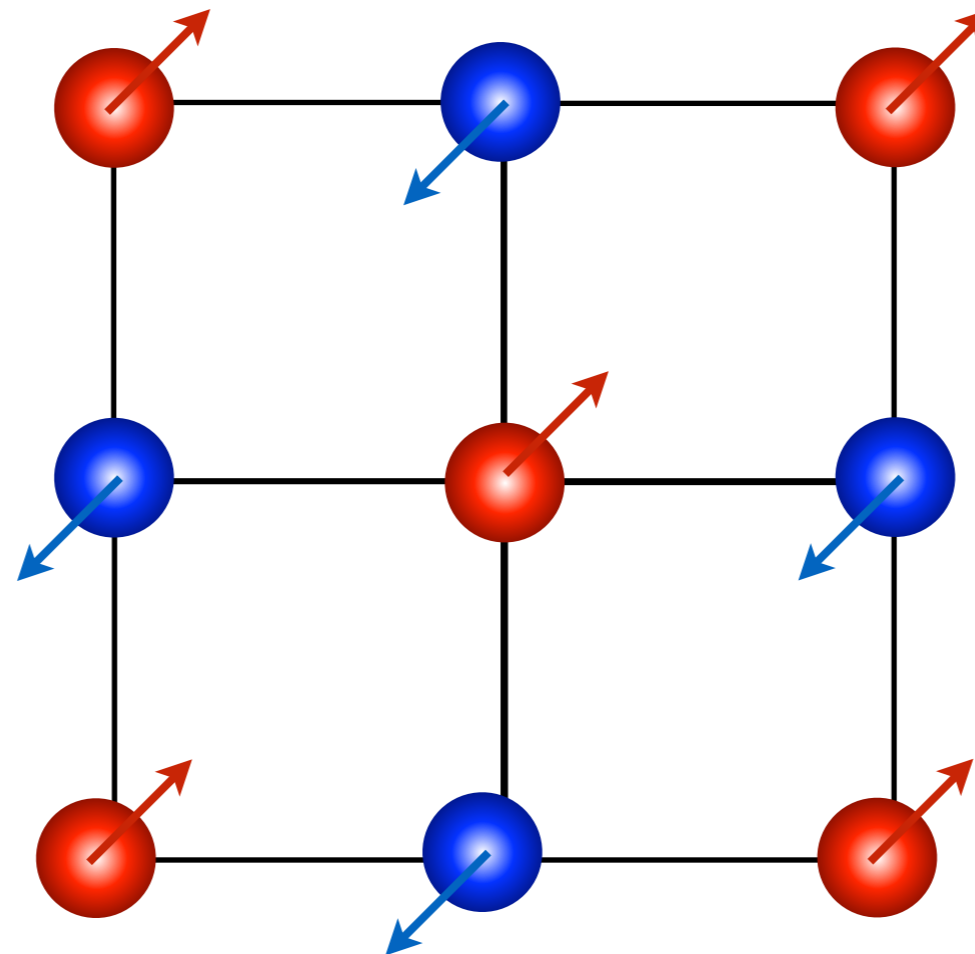
The first class of quantum materials are generally magnetic, and in many cases, their novel properties are intimately related with their magnetism...



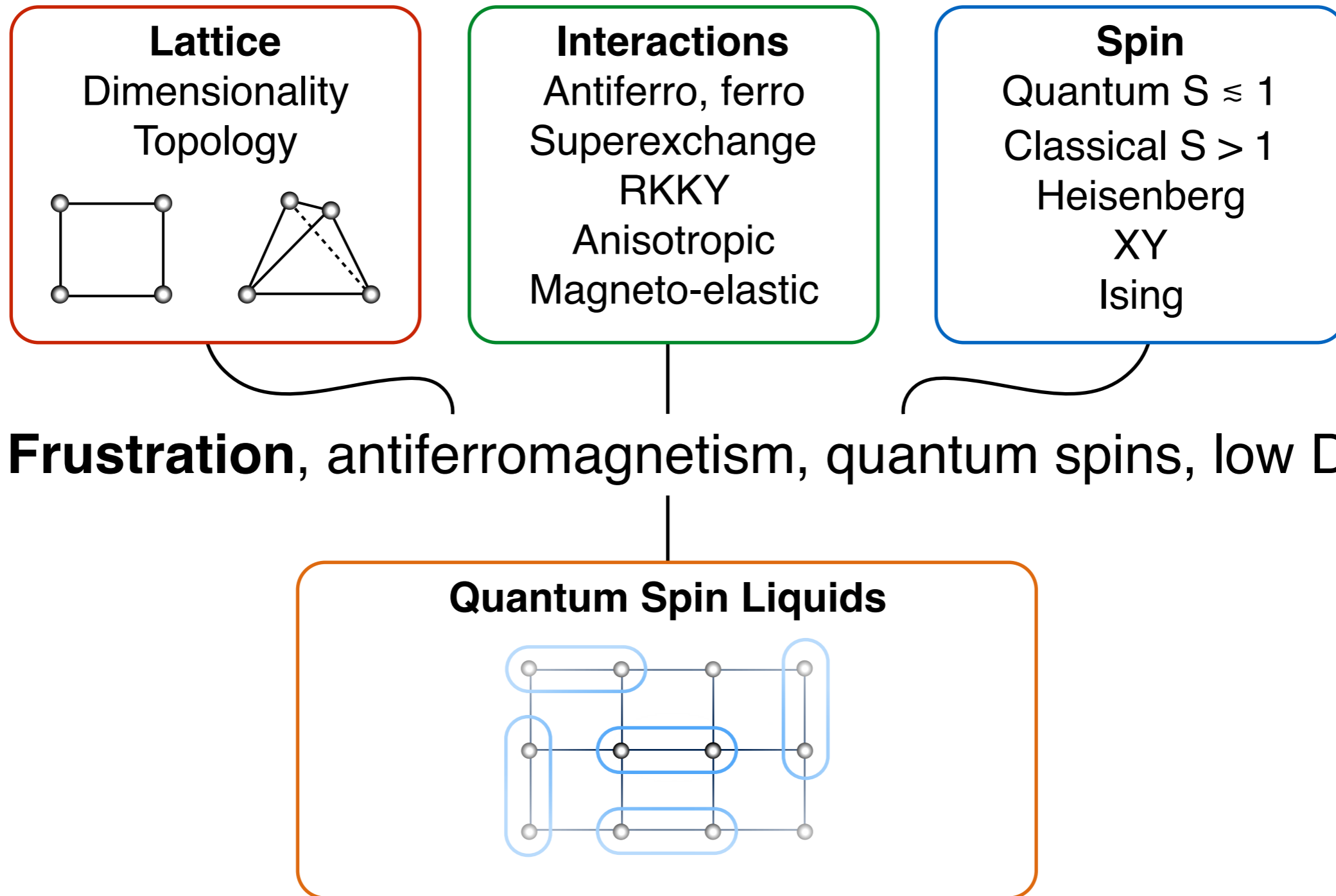
Basov and Chubukov Nature Phys. **7** 272

However, these are complex systems with spin, charge, and orbital degrees of freedom, which makes them difficult to deal with theoretically.

Can exotic states be achieved without chemical doping (and all the ensuing complexity)? Yes, in magnetic systems...

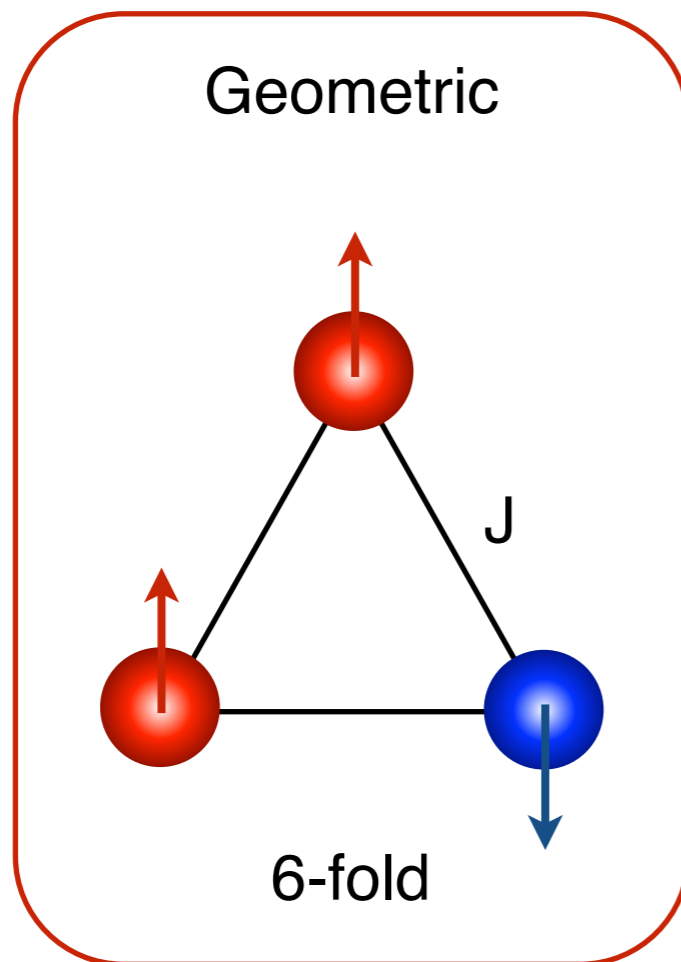


... however, most of these order at high temperature. Need to find a way to enhance quantum fluctuations and destroy conventional magnetic order.

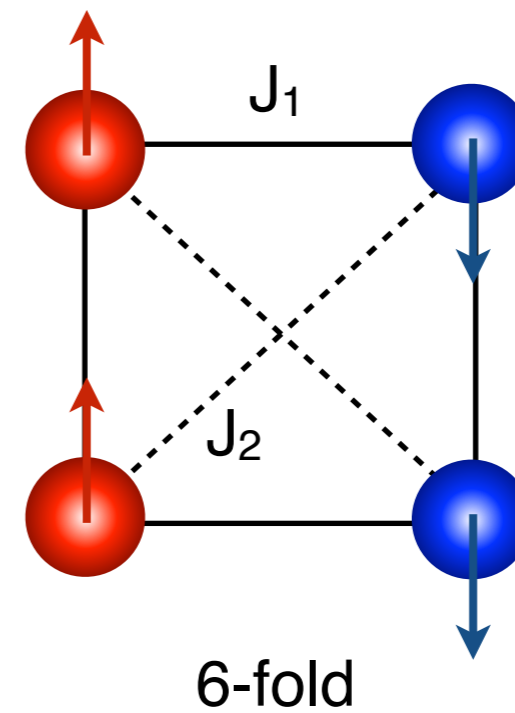


- Magnetic frustration
- QSL in the kagome lattice antiferromagnet
  - Ground state
  - Excitations - inelastic neutron scattering
- Neutron scattering on kagome lattice materials
  - Overview of  $\text{Cu}^{2+}$  minerals
  - Herbertsmithite: our best shot at a QSL so far
  - Volborthite: orbital order and trimerization
  - $\text{KCu}_3\text{As}_2\text{O}_7(\text{OD})_3$ : multiferroicity in a material far from the QSL limit
- Conclusion

# What is frustration?

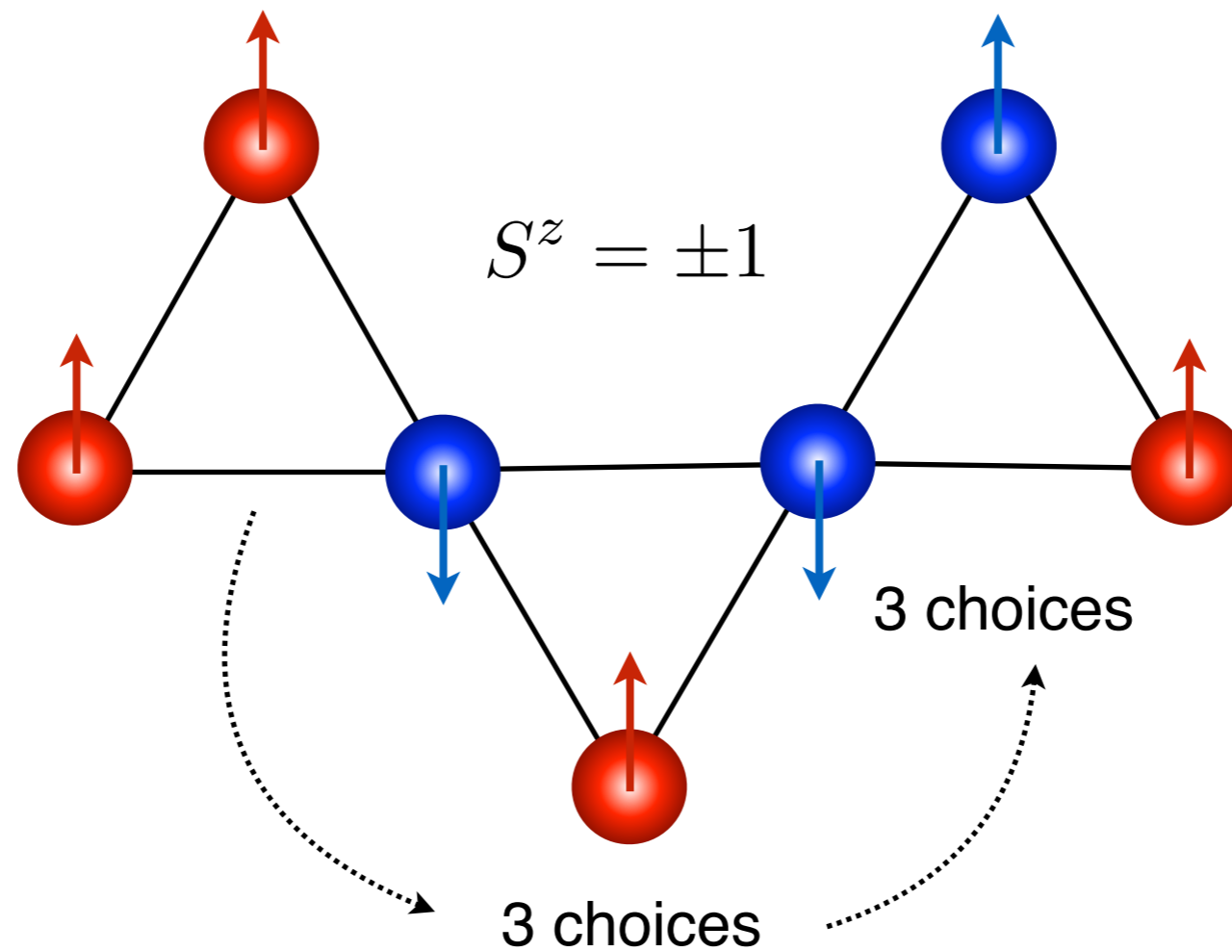


Non-geometric



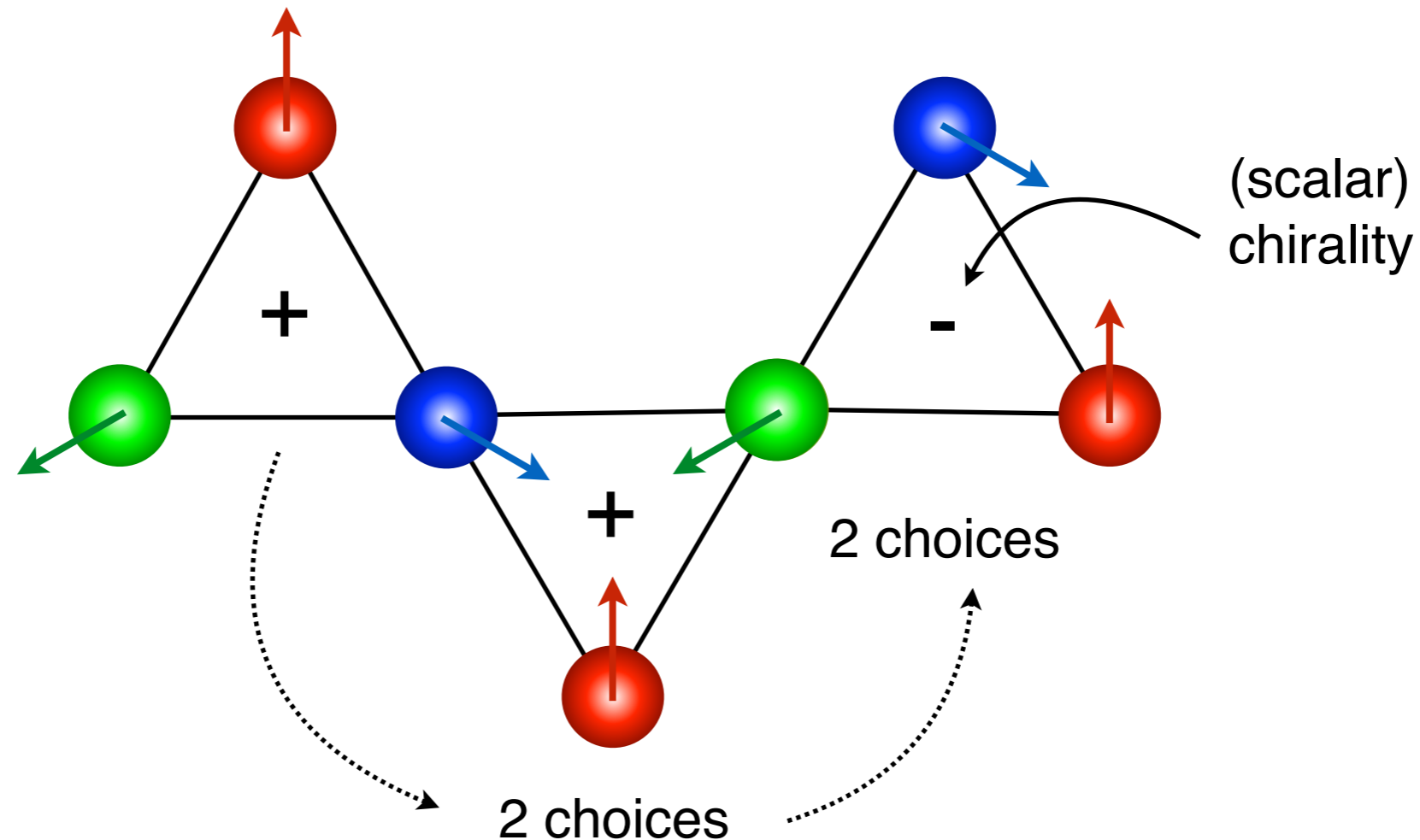
$$S^z = \pm 1$$

$$\mathcal{H} = J \sum_{i,j} S_i^z S_j^z$$



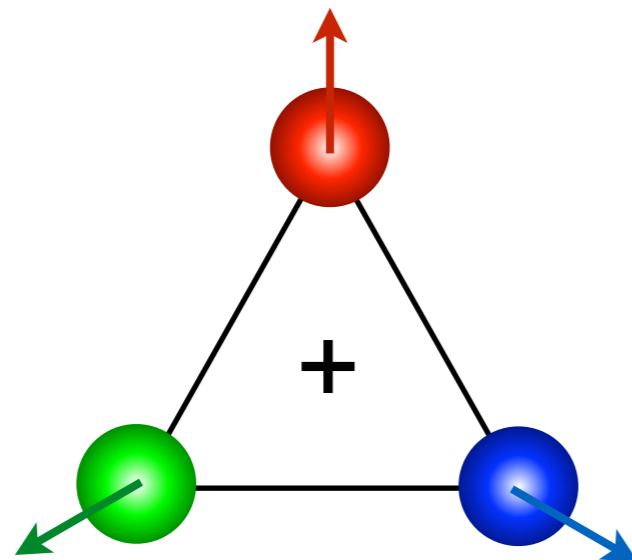
Husimi and Syozi Prog. Theor. Phys. **5** 117 (1950)





In both cases, huge degeneracy in GS (sometimes lifted by fluctuations)  
Large barrier to conventional magnetic order

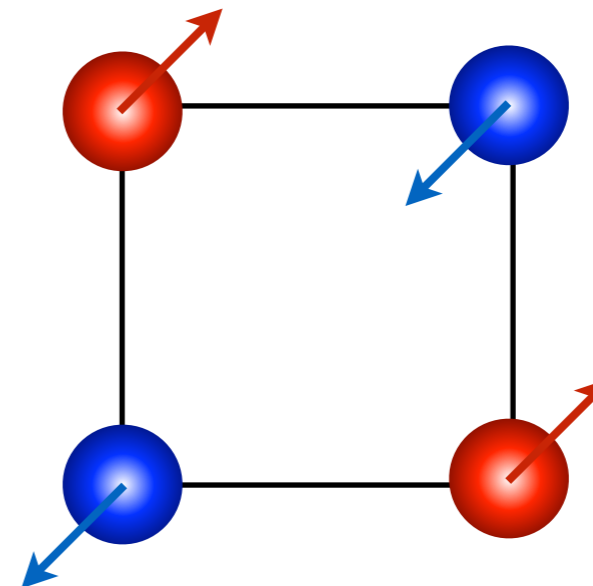
Frustrated



$120^\circ$

$$\langle E_{ij} \rangle = JS^2 \cos \theta = -\frac{JS^2}{2}$$

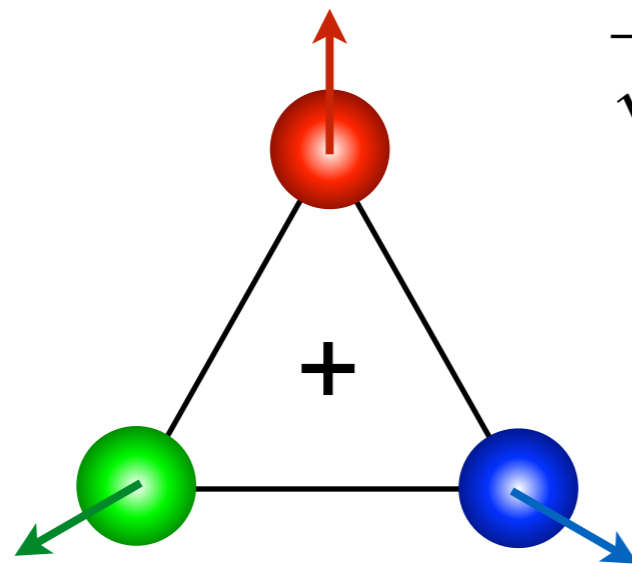
Unfrustrated



“Néel”

$$\langle E_{ij} \rangle = -JS^2$$

Classical (vector)

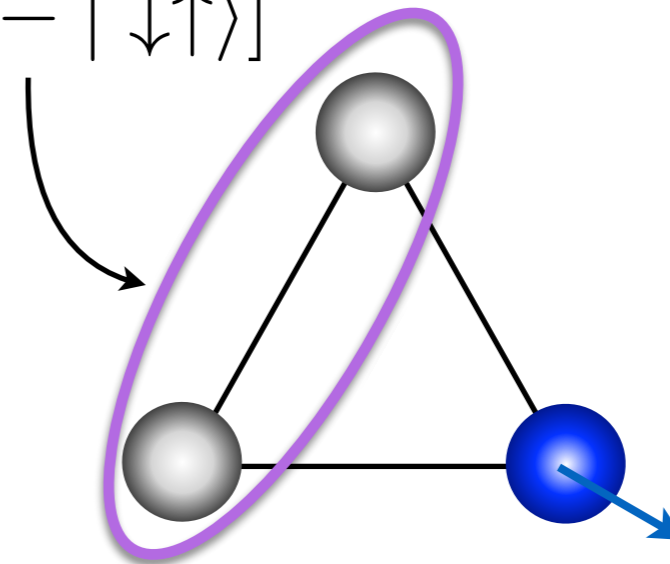


$120^\circ$

$$\langle E_{ij} \rangle = JS^2 \cos \theta = -\frac{JS^2}{2}$$

Quantum (operator)

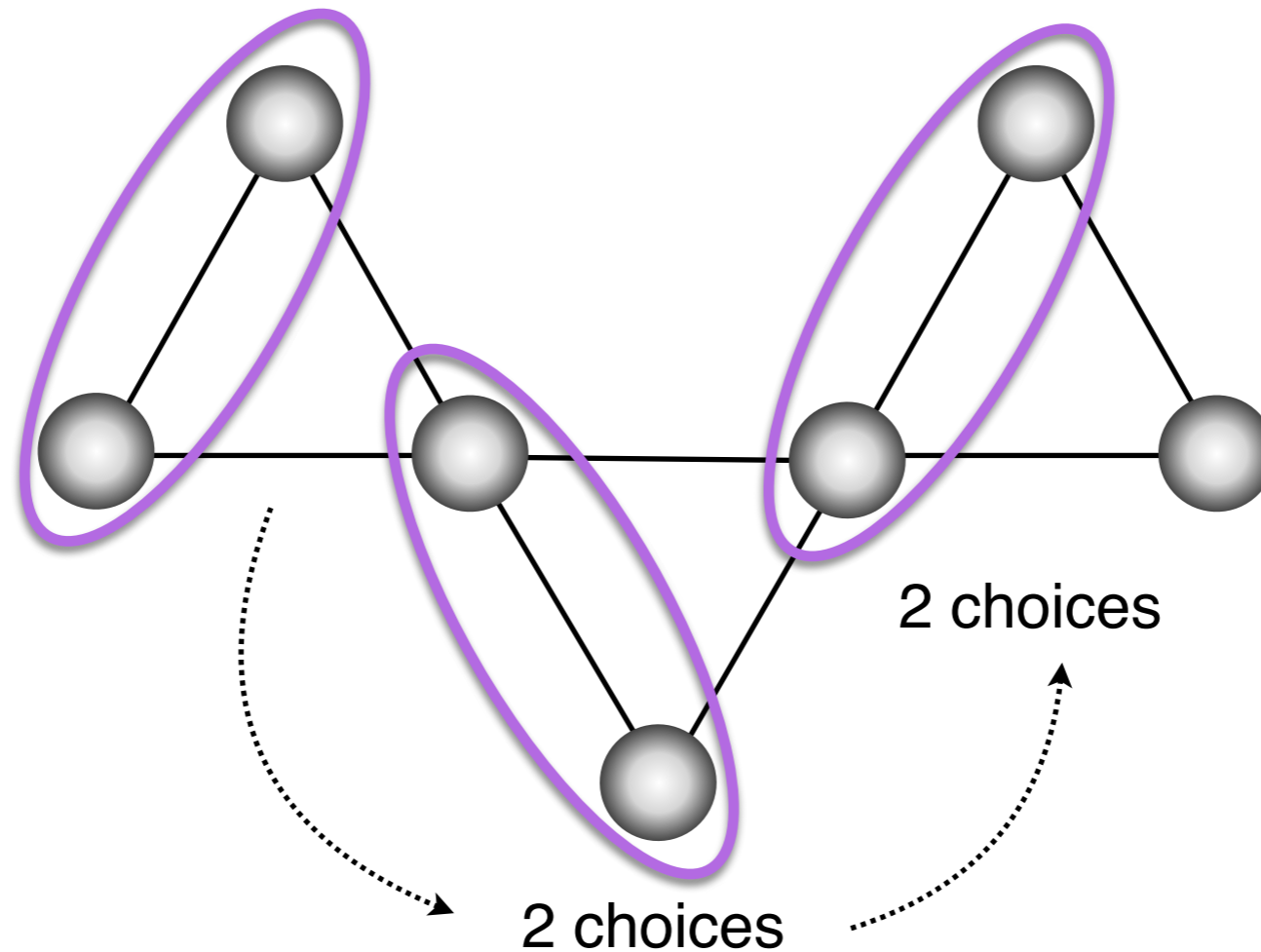
$$\frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$



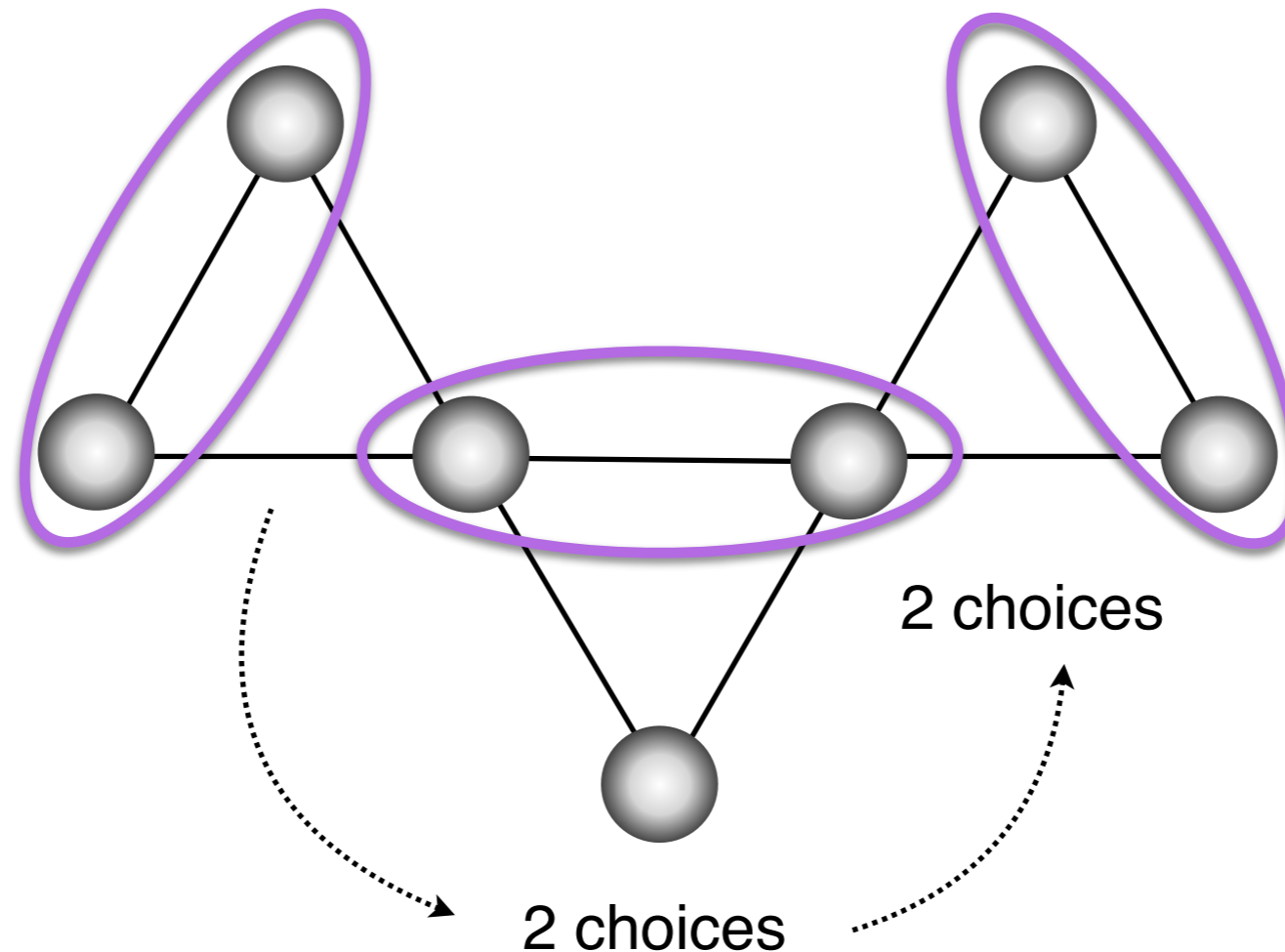
“Valence bond”

$$\langle E_{ij} \rangle = -\frac{JS(S+1)}{3}$$

Just like in the classical case, there is an exponentially large number of dimer coverings of the lattice...

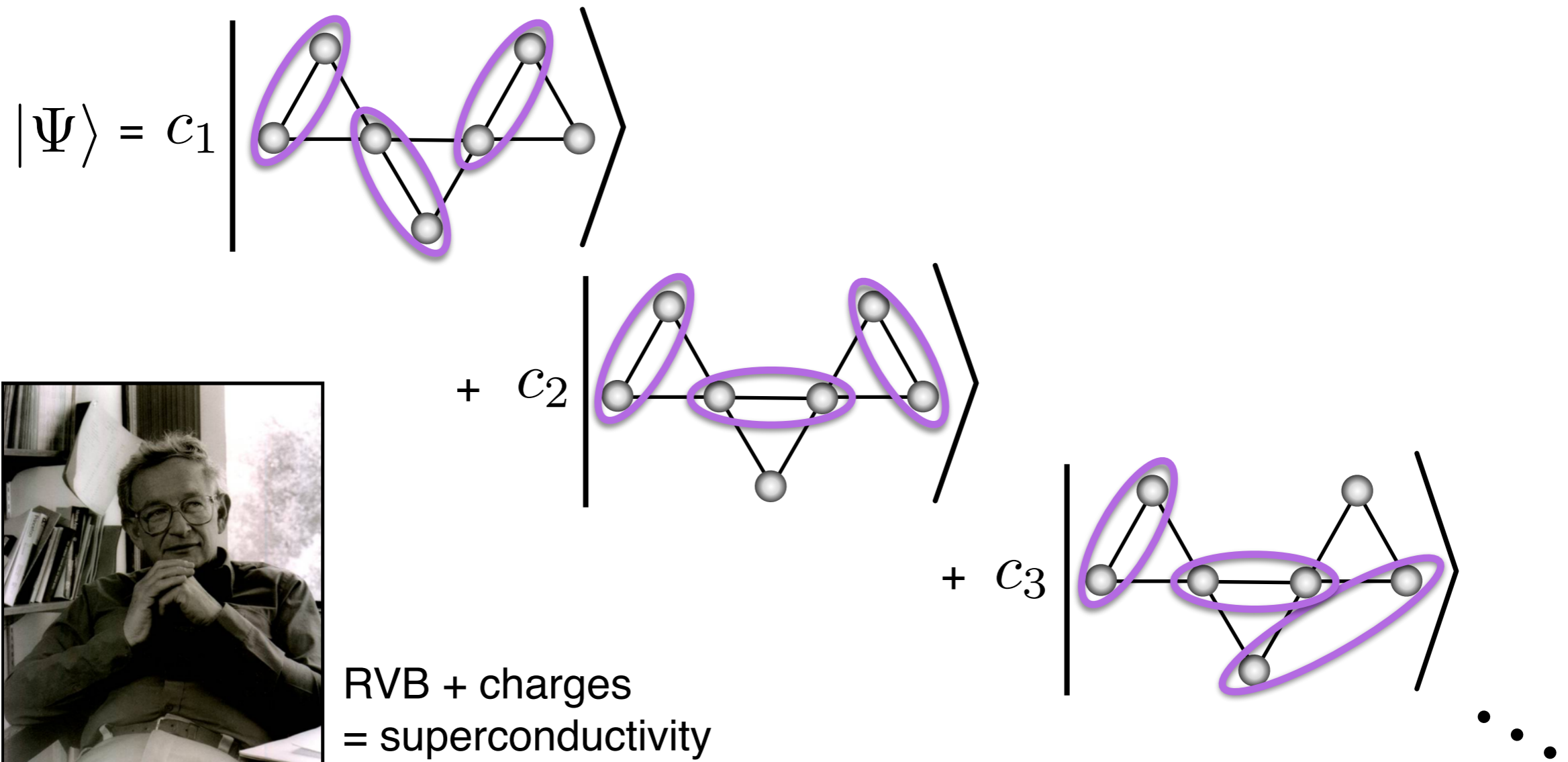


Just like in the classical case, there is an exponentially large number of dimer coverings of the lattice...



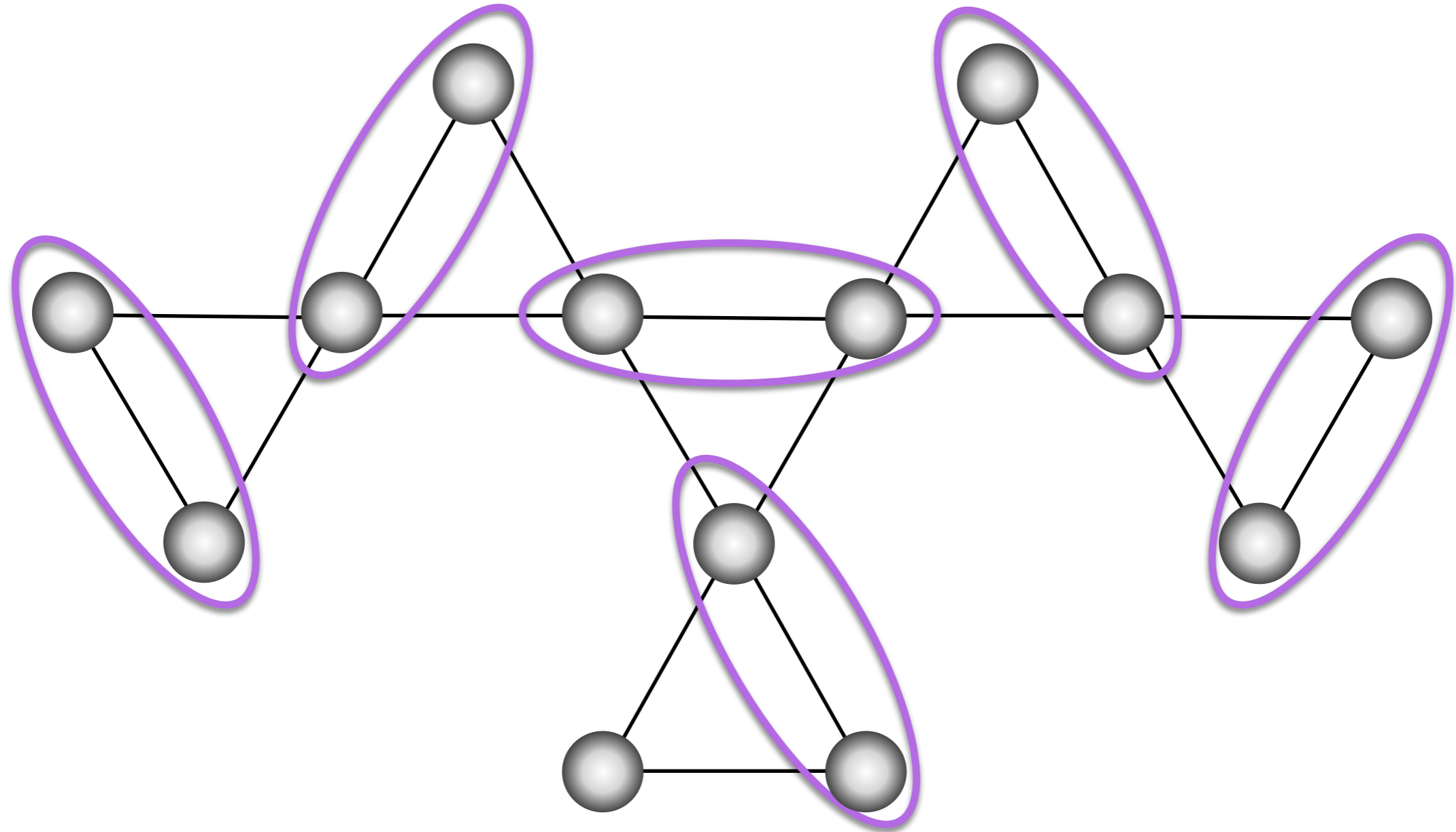
# Resonating Valence bond

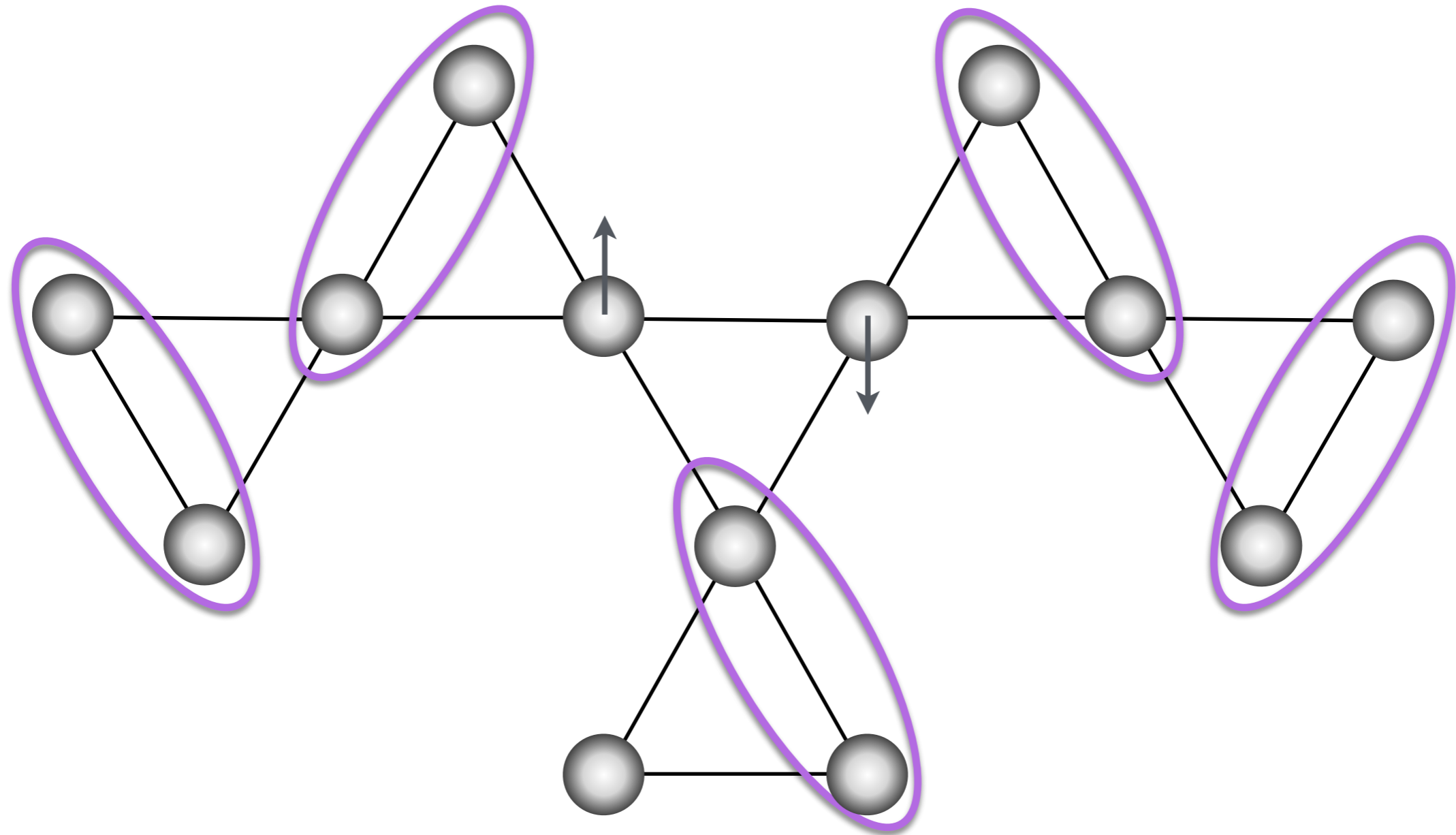
The system can gain energy by fluctuating between these configurations, as well as ones with longer-ranged bonds:



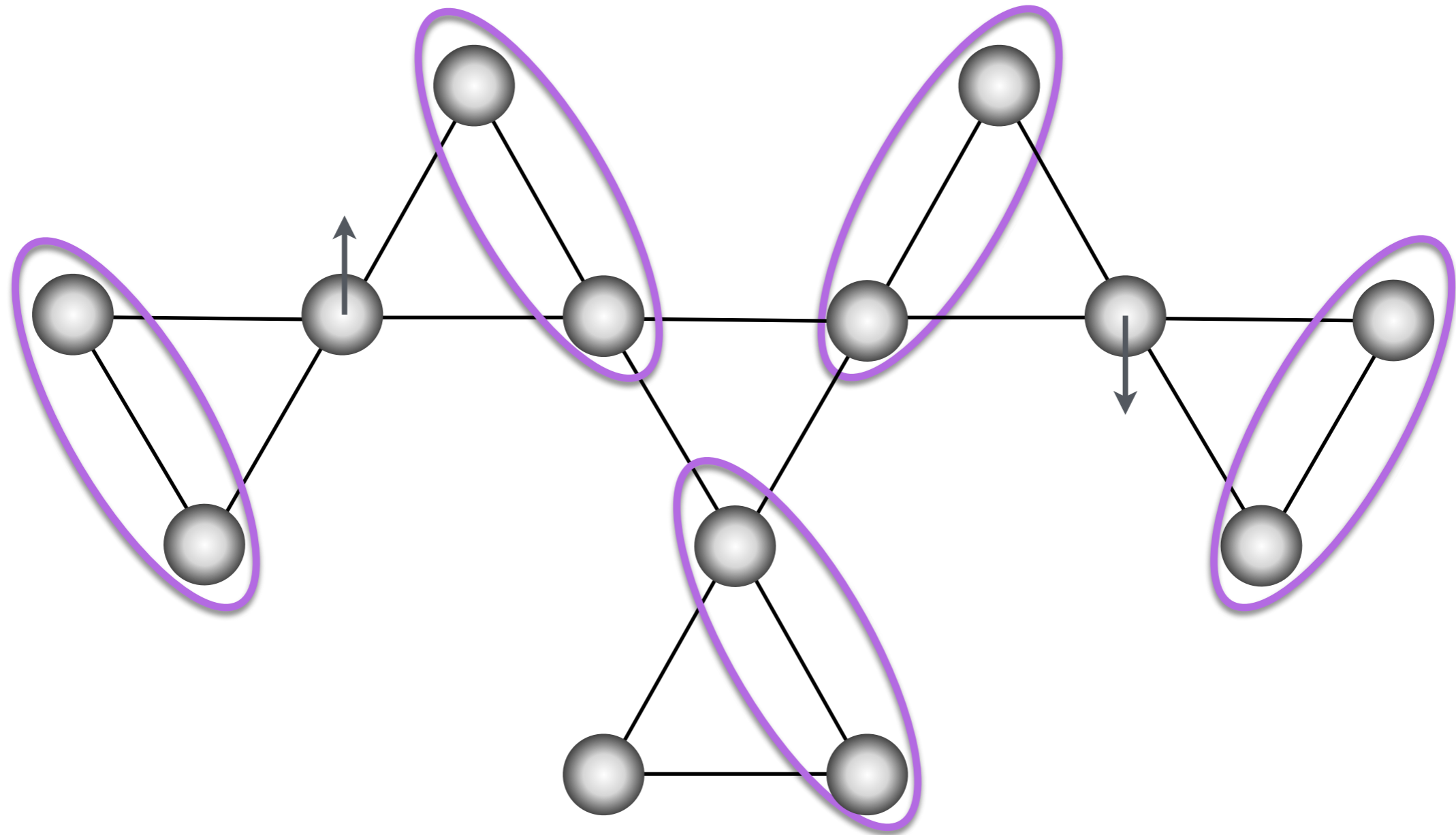
Anderson Science **235** 1196

If one of the singlets is broken,

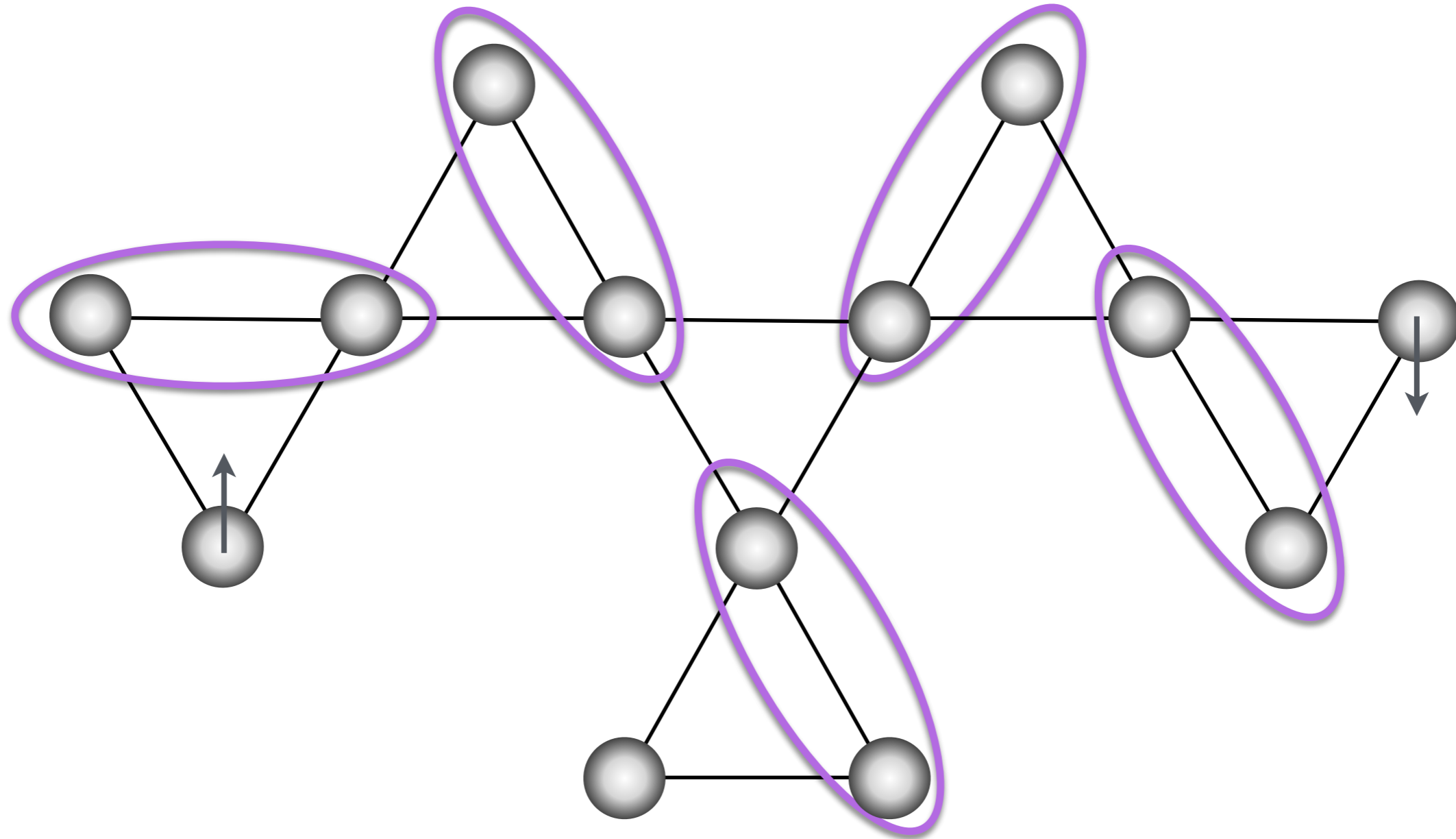




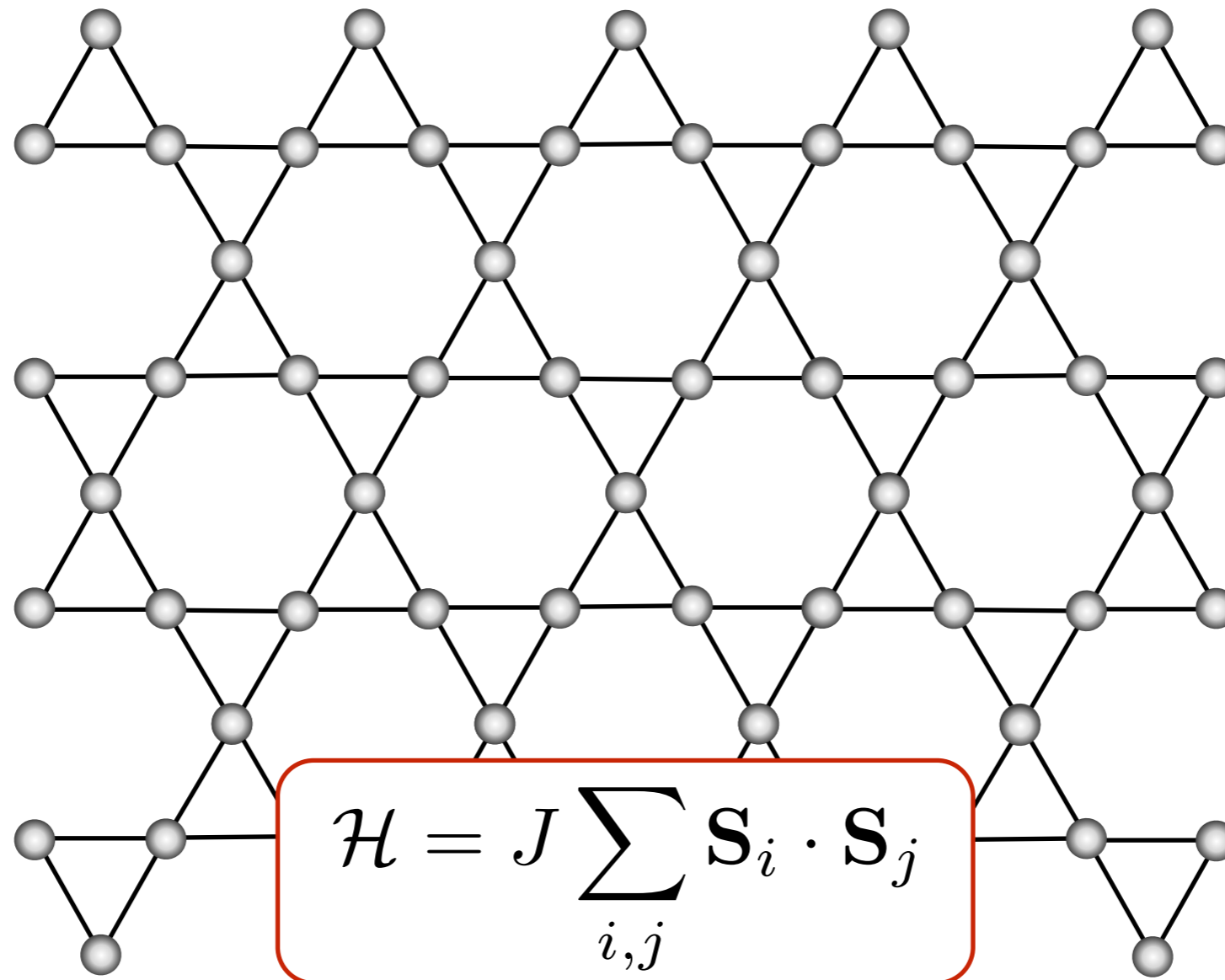




Fractionalized, (potentially) deconfined quasiparticles...



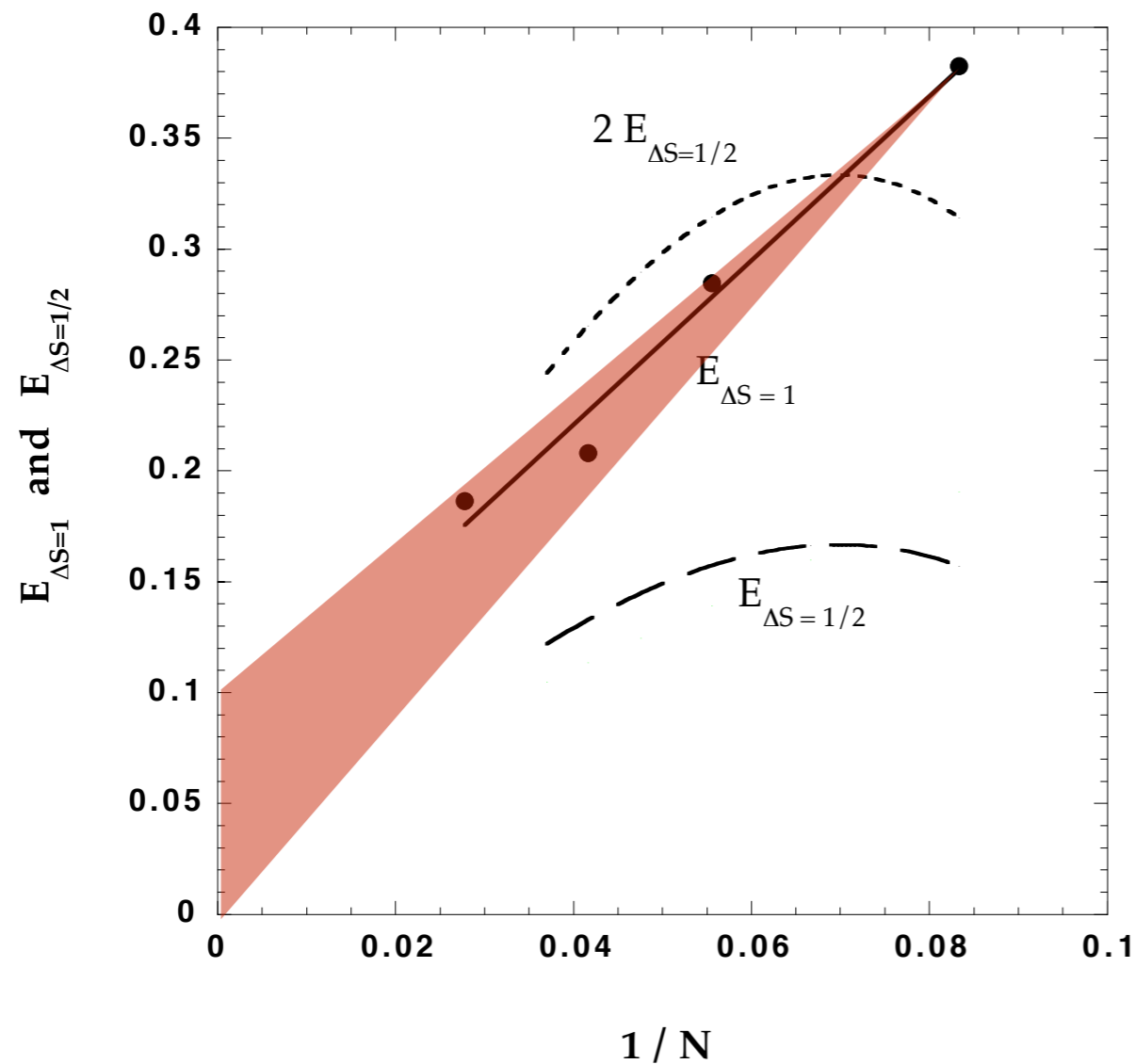
The  $S = 1/2$  kagome lattice antiferromagnet is **THE** model of frustration in 2D:



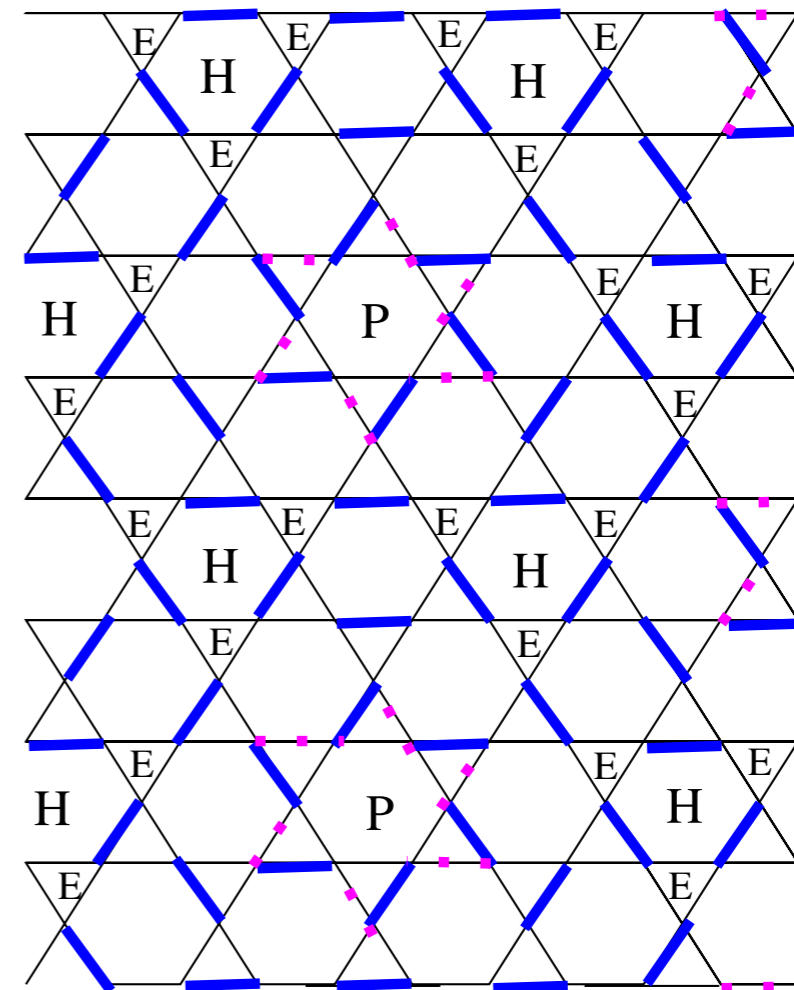
Is its ground state the long-sought RVB state?



Its ground state was initially proposed to be a gapless QSL, then a gapped VBS:

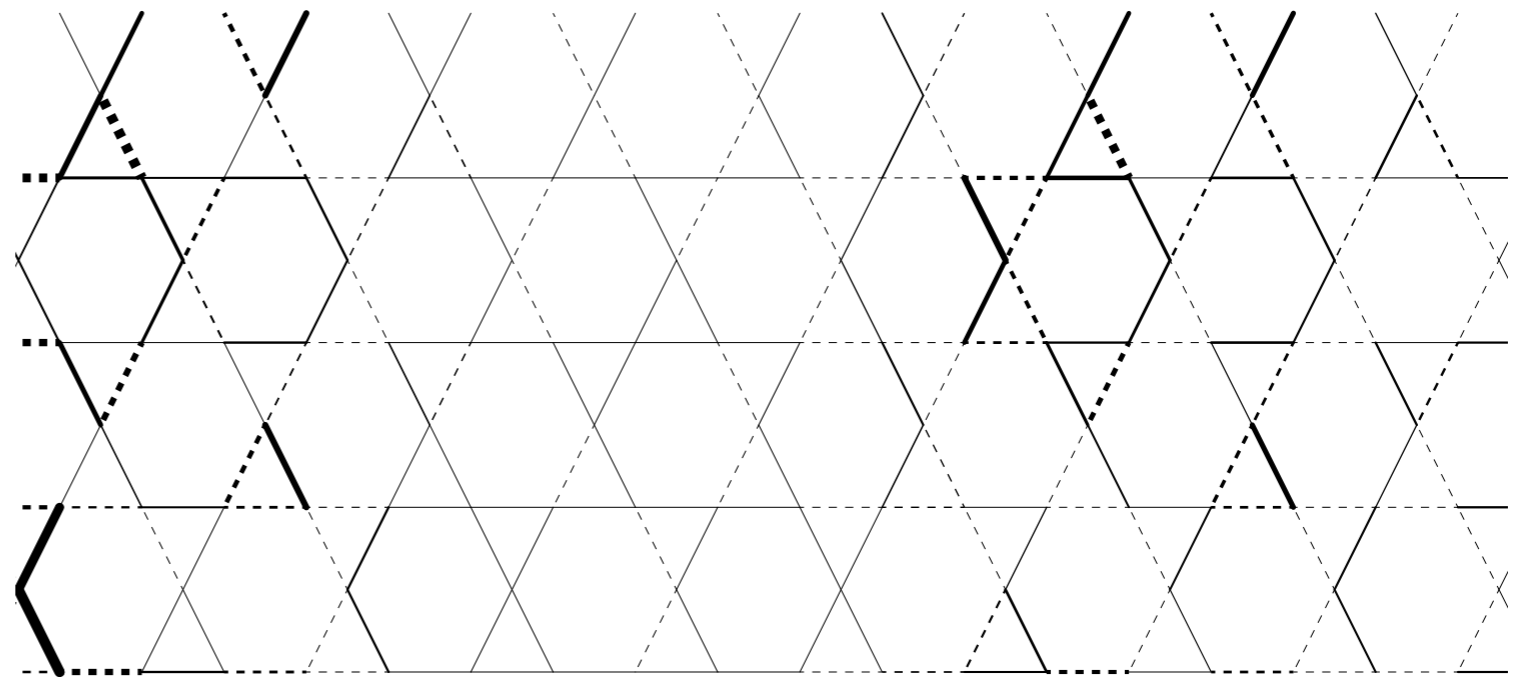
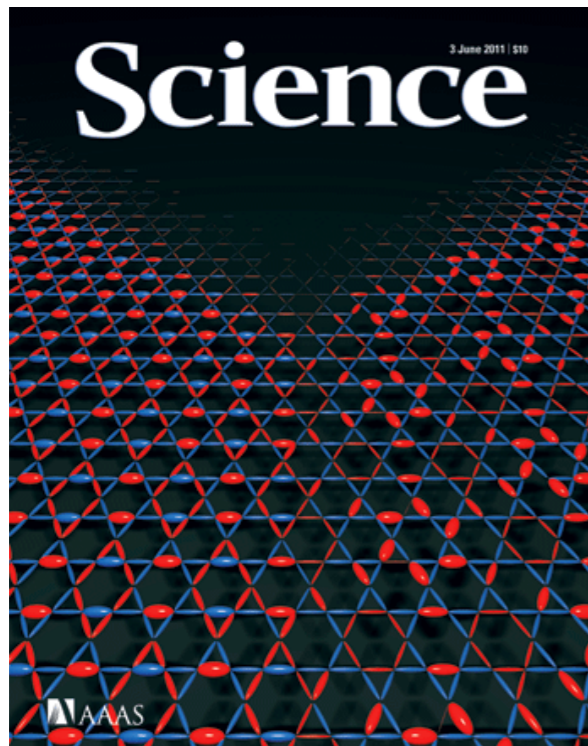


Lecheminant et. al. PRB **56** 2521  
Waldtmann et. al. EPJB **2** 501



Singh and Huse PRB **76** 180407 (R)

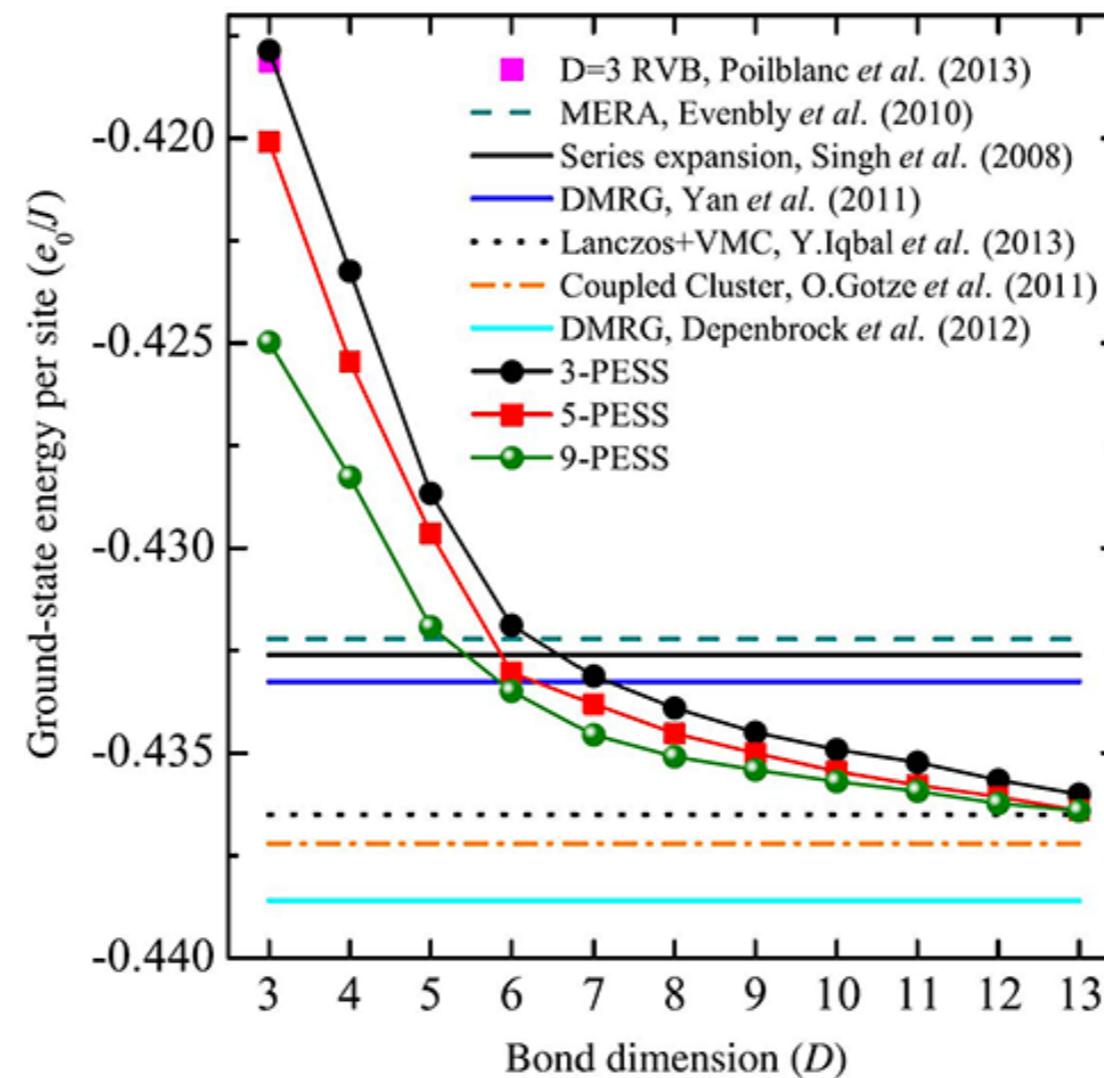
In 2011, state-of-the art DMRG simulations suggested strongly fluctuating SL with weak dimer correlations:



Yan, Huse and White *Science* **332** 1173

Still considerable debate, however: not  $SU(2)$  (uniform RVB) — either  $U(1)$  or  $Z_2$  — in the RVB picture, these correspond to different weightings of bonds

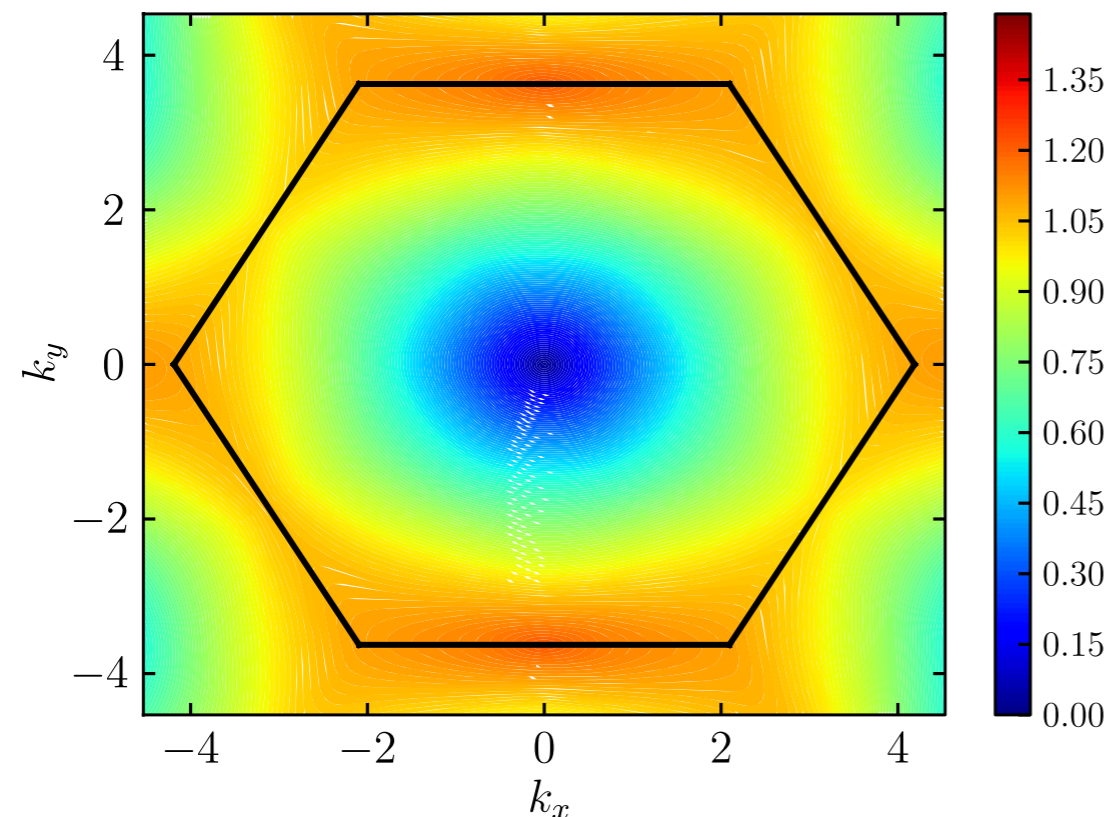
States separated by minute energies (near a QCP), so interactions beyond  $J$  can drastically alter the ground state. These are inevitable in real systems...



Liao et. al. PRL **118** 137202

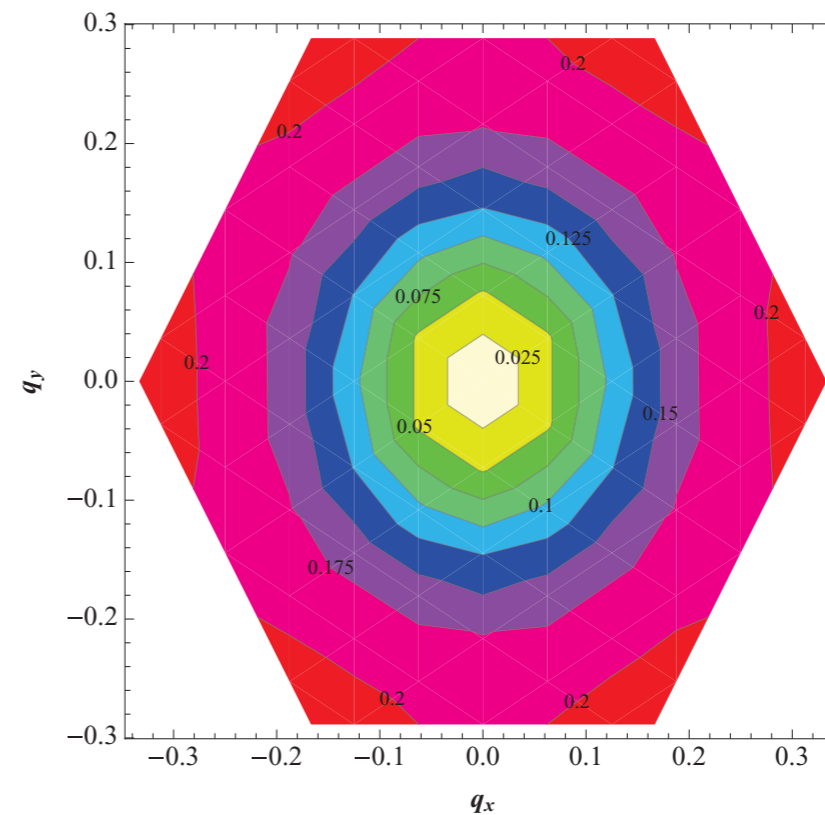
Regardless what the actual ground-state is, the static structure factors are nearly identical:

### $Z_2$ QSL $S(Q)$



Depenbrock et. al. PRL **109** 067201

### $U(1)$ QSL $S(Q)$



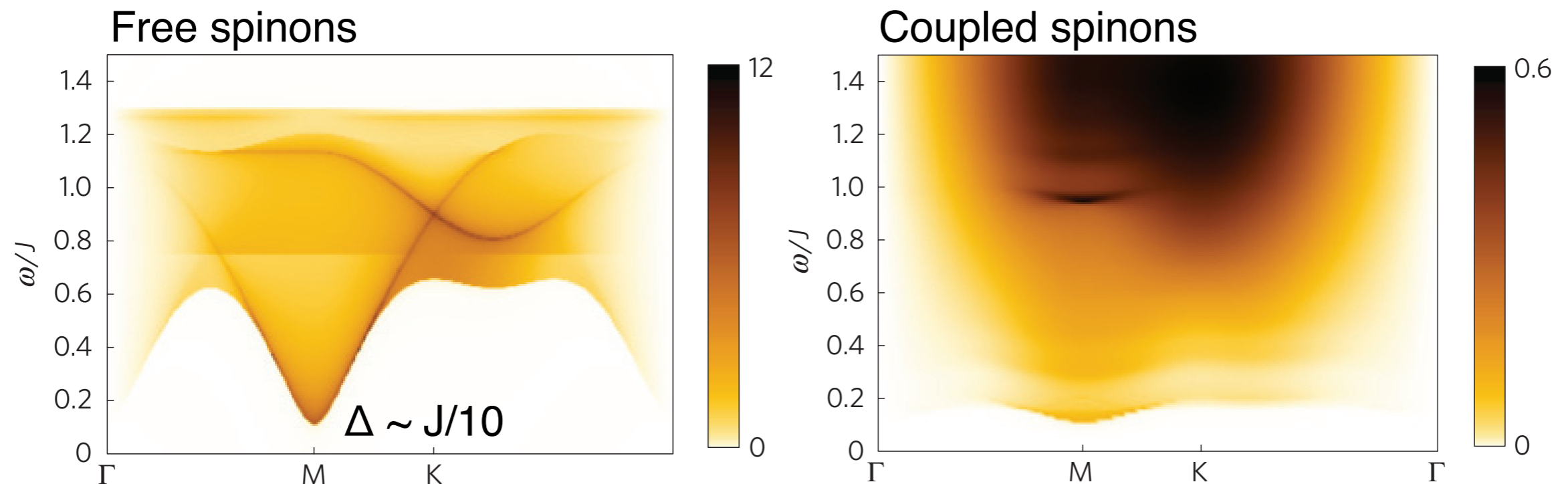
Iqbal et. al. **87** 060405

However, the  $Z_2$  QSL is associated with a small gap  $\Delta$  in the excitation spectrum, while the  $U(1)$  should be gapless...



Non-interacting spinon limit: strongly dispersing continuum (one neutron, two spinons). Introduce interactions between spinons and other excitations: broaden.

## $Z_2$ QSL $S(Q,\omega)$



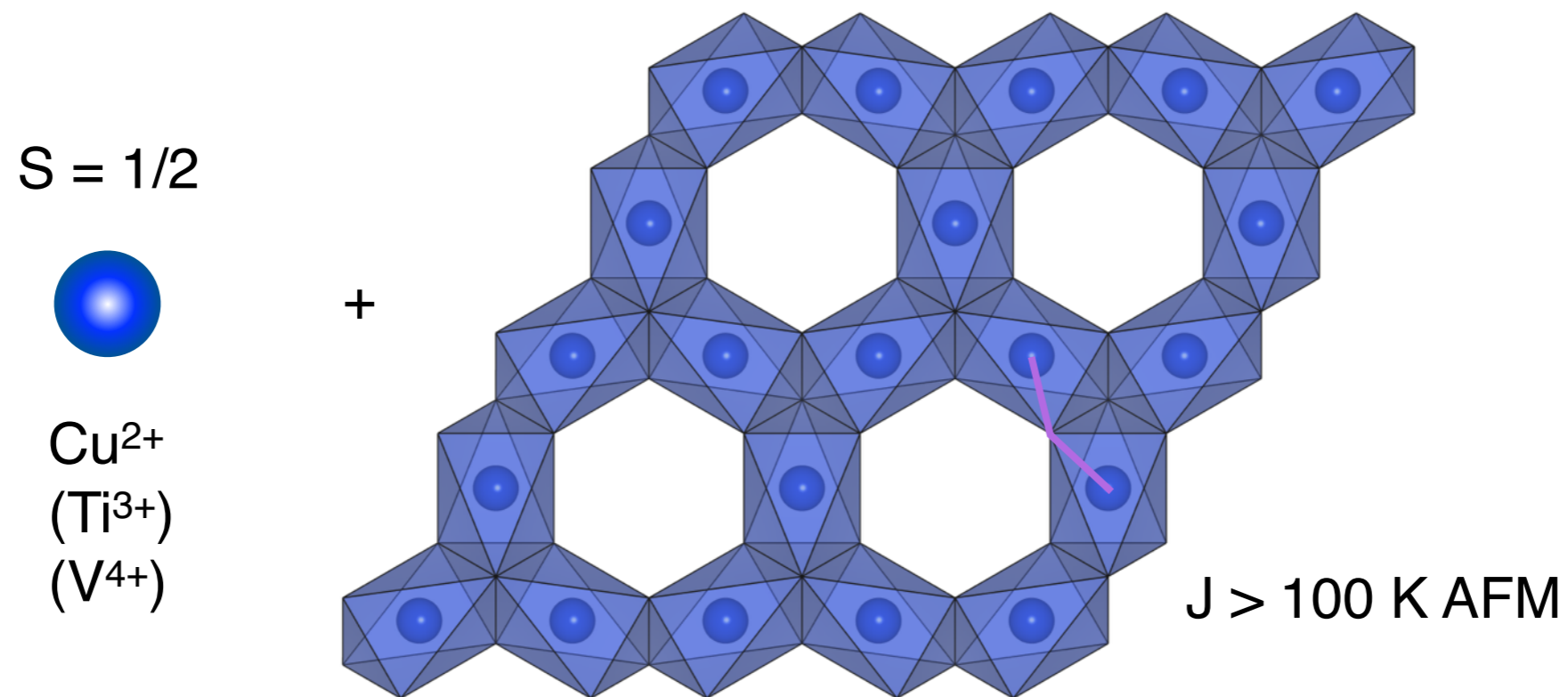
Punk et. al. Nature Phys. **10** 289; Dodds et. al. PRB **88** 224413

Spinons coupled to visons -  
topological singlet excitations.

# Cu<sup>2+</sup> minerals as realisations of the kagome lattice antiferromagnet

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How can we realize a kagome lattice material? If we consider inorganic materials with TM octahedra...

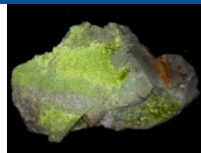



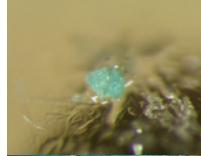



Most materials studied so far contain some variation on this motif...

Sometimes nature gets in the way...

## Hiking in Norway



Material	SG	$\theta$	Order	Reference(s)	
Volborthite $\text{Cu}_3\text{V}_2\text{O}_7(\text{OH})_2 \cdot 2\text{H}_2\text{O}$	$C2/m$	-115 K	1 K	Z. Hiroi <i>et al.</i> , JPSJ <b>70</b> , 3377 M. Yoshida <i>et al.</i> , PRL <b>103</b> , 077207 G. J. Nilsen <i>et al.</i> , PRB <b>84</b> , 172401	
Herbertsmithite $\beta\text{-Cu}_3\text{Zn}(\text{OH})_6\text{Cl}_2$	$R\bar{3}m$	-240 K	< 50 mK	M. P. Shores <i>et al.</i> JACS <b>127</b> , 13462 de Vries <i>et al.</i> PRL <b>103</b> , 237201 T. Han <i>et al.</i> Nature <b>492</b> , 406	
$\text{KCu}_3\text{As}_2\text{O}_7(\text{OD})_3$	$C2/m$	+13.4 K	7 K	Y. Okamoto <i>et al.</i> JPSJ <b>81</b> , 033707	
Bayldonite $\text{PbCu}_3(\text{AsO}_4)_2(\text{OH})_2$	$C2/c$	+49 K	< 2 K ?	Unpublished (H. Ishikawa, Y. Okamoto)	
Vesignieite $\text{BaCu}_3(\text{VO}_4)_2(\text{OH})_2$	$C2/m$	-59 K	7 K	Y. Okamoto <i>et al.</i> , JPSJ <b>78</b> , 033701 M. Yoshida <i>et al.</i> , JPSJ <b>82</b> , 013702	
Edwardsite $\text{Cu}_3\text{Cd}_2(\text{SO}_4)_2(\text{OH})_6 \cdot 4\text{H}_2\text{O}$	$P2_1/c$	- 50 K	4.3 K	H. Ishikawa <i>et al.</i> JPSJ <b>82</b> , 063710	
Kapellasite $\alpha\text{-Cu}_3\text{Zn}(\text{OH})_6\text{Cl}_2$	$R\bar{3}m$	0 K	< 50 mK	Colman <i>et al.</i> , Chem. Mater <b>20</b> 6897 Fåk <i>et al.</i> PRL, <b>109</b> 137208	
$\text{Cu}_3\text{Zn}(\text{OH})_6(\text{SO}_4)$	$P2_1/a$	- 79 K	< 50 mK	Y. Li <i>et al.</i> , cond-mat:1310.2795	

What is the nature of the QSL in  
Herbertsmithite?

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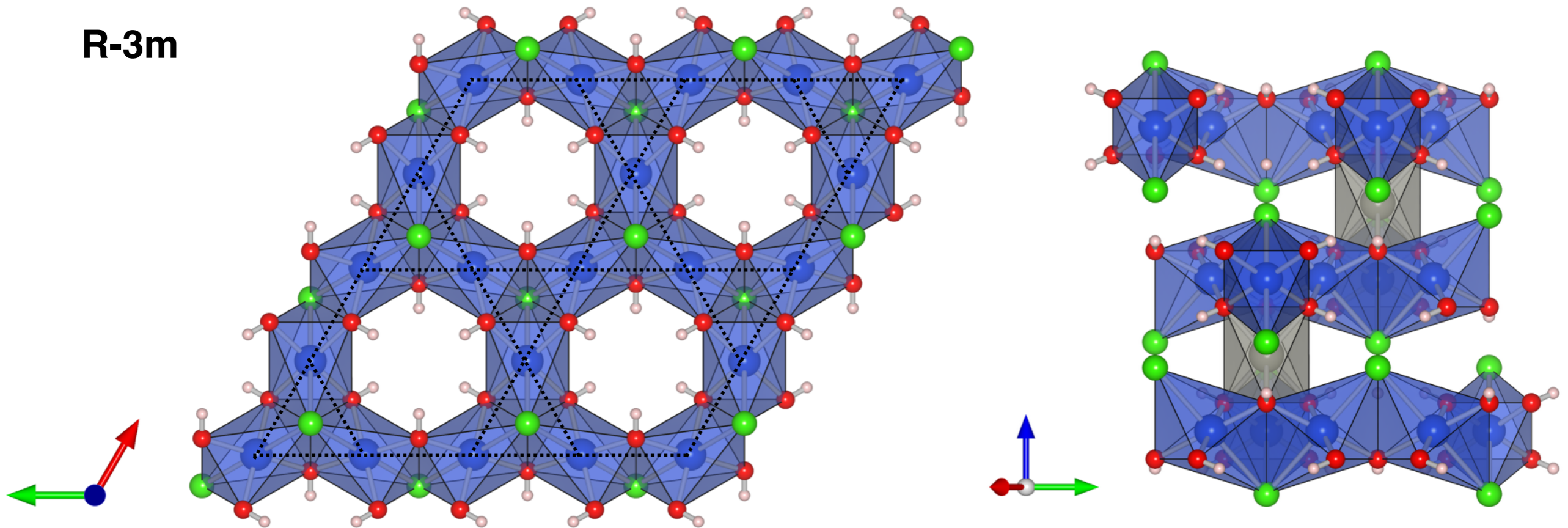
# Herbertsmithite, $\text{Cu}_3\text{Zn}(\text{OH})_6\text{Cl}_2$



Science & Technology  
Facilities Council

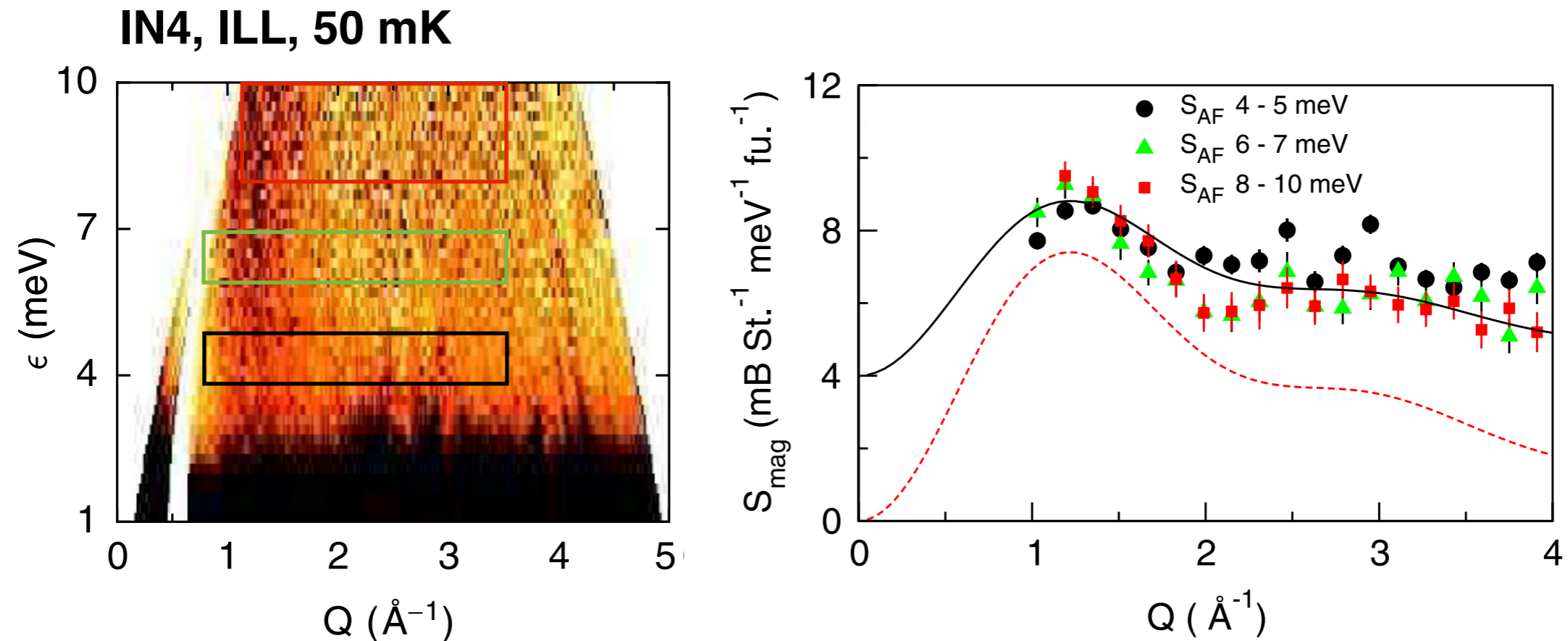
$\theta = -240 \text{ K}$ ,  $J = 180 \text{ K}$ ,  $T_N < 50 \text{ mK}$

R-3m



M. P. Shores et. al. JACS **127** 13462

High-energy  $S(Q, \omega)$  consistent with very short-range correlations even at 50 mK!



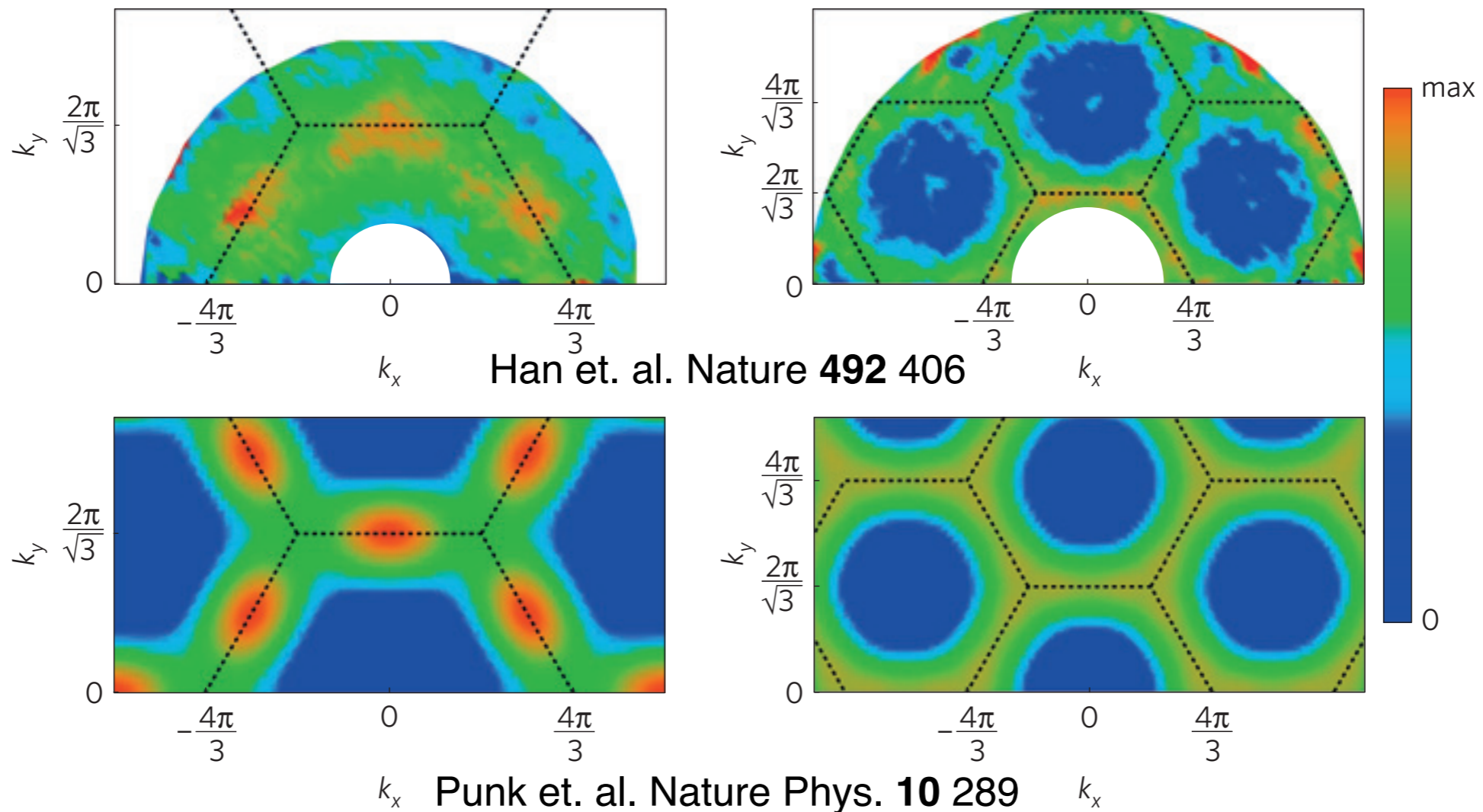
M. A. de Vries GJN et. al. PRL **103** 237201

No sign of gap down to  $\sim 3$  meV.



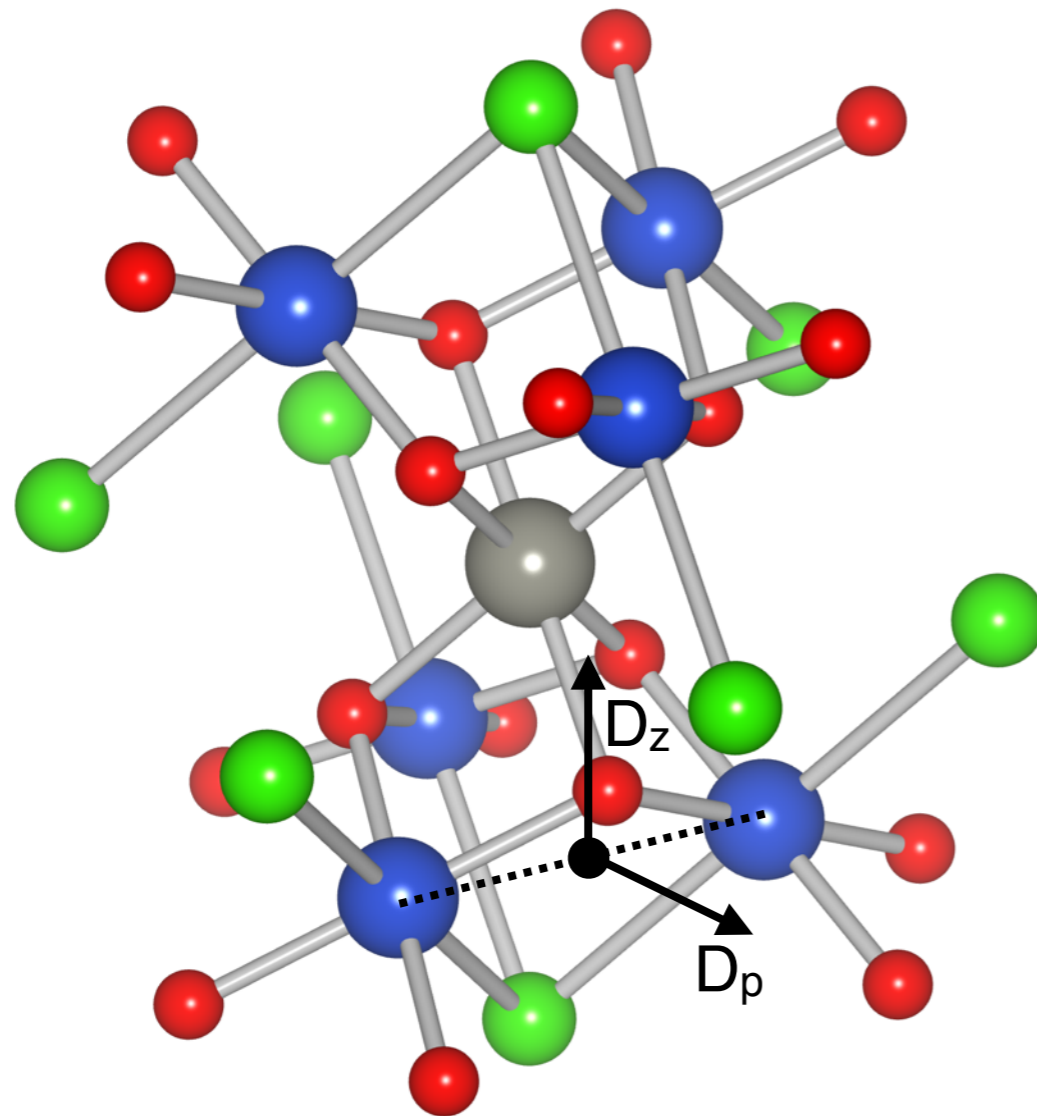
High-energy  $S(Q, \omega)$  consistent with very short-range correlations even at 50 mK!

## MACS, NIST, 1.6 K



Calculation for  $Z_2$  spin liquid with spinon-vison interactions agrees qualitatively!

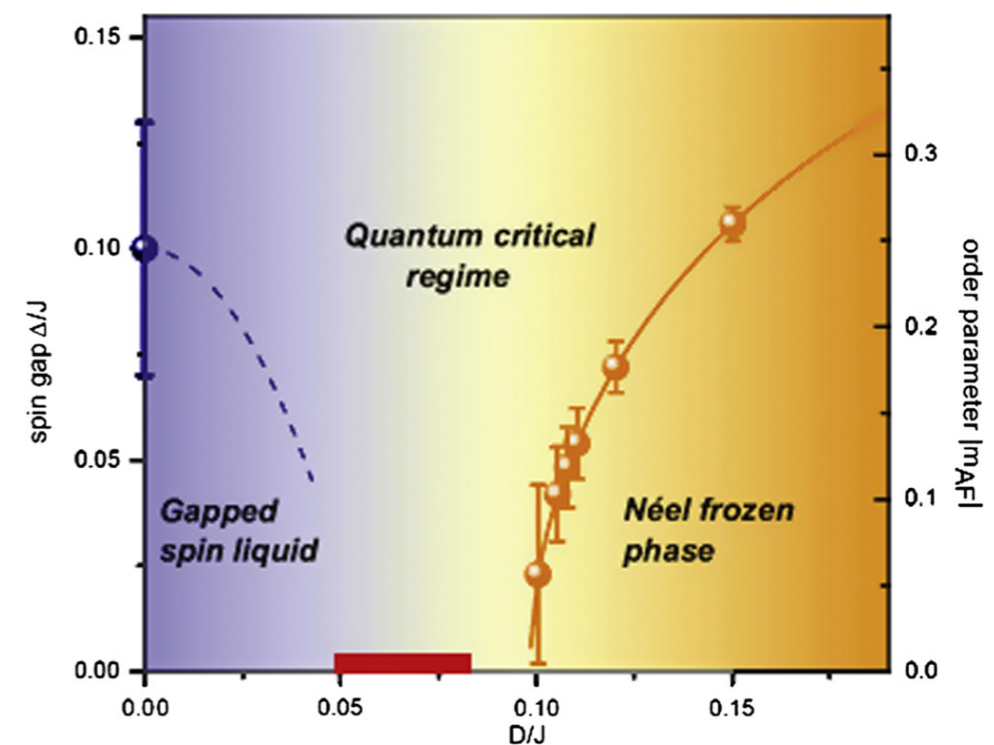
These look like encouraging signs of a QSL, but there are a few caveats...



Lack of inversion centre means  
Dzyaloshinskii-Moriya allowed:

$$\mathbf{D} \cdot \mathbf{S}_i \times \mathbf{S}_j$$

ESR suggests  $D_z = 15 \text{ K} \sim 0.08 \text{ J}$

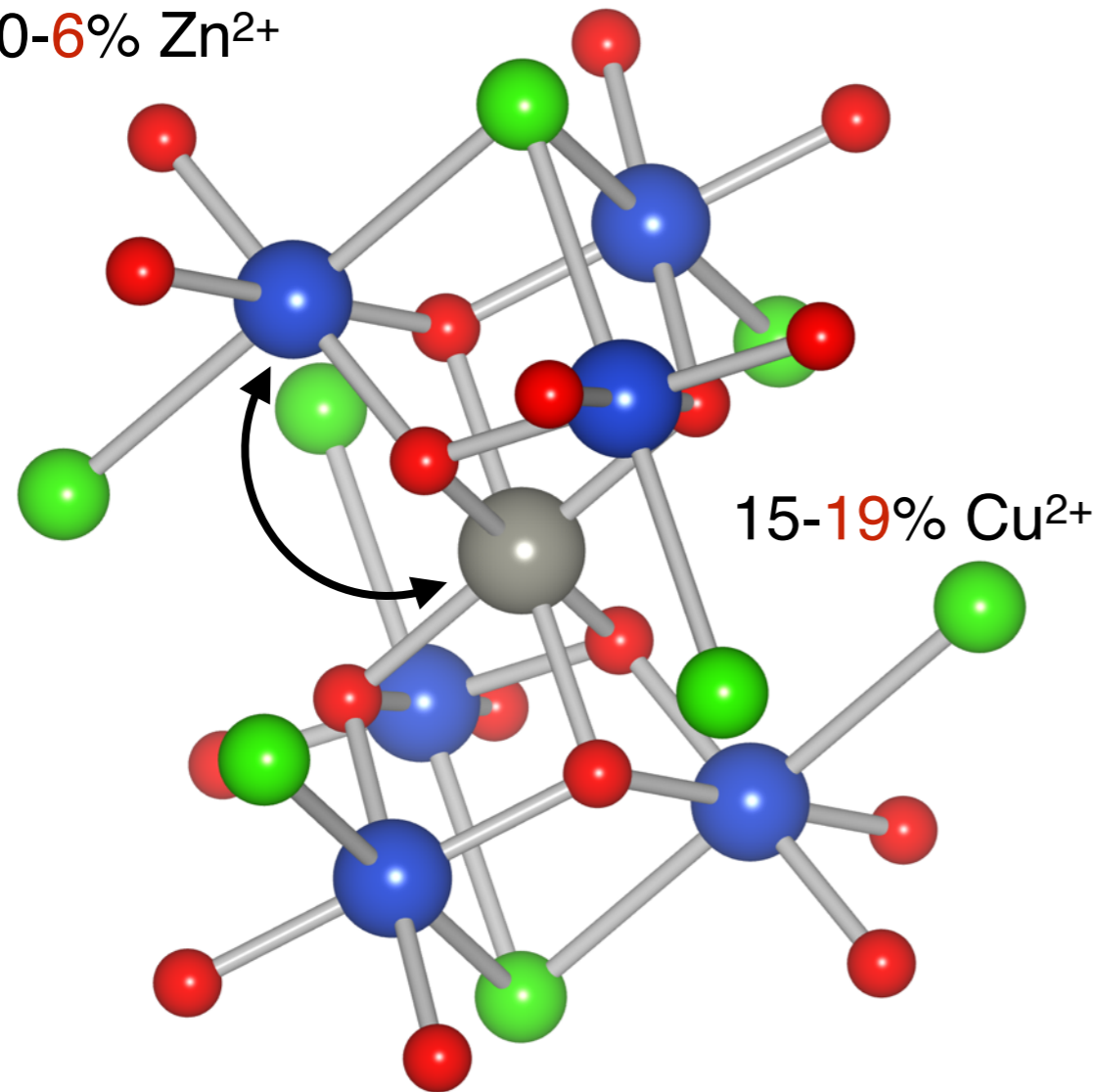


Zorko et. al. PRL **101** 026405; Bert et. al. Reverts Phys. **37** 4; Cepas et. al. PRB **78** 140405

# Herbertsmithite: a true QSL?

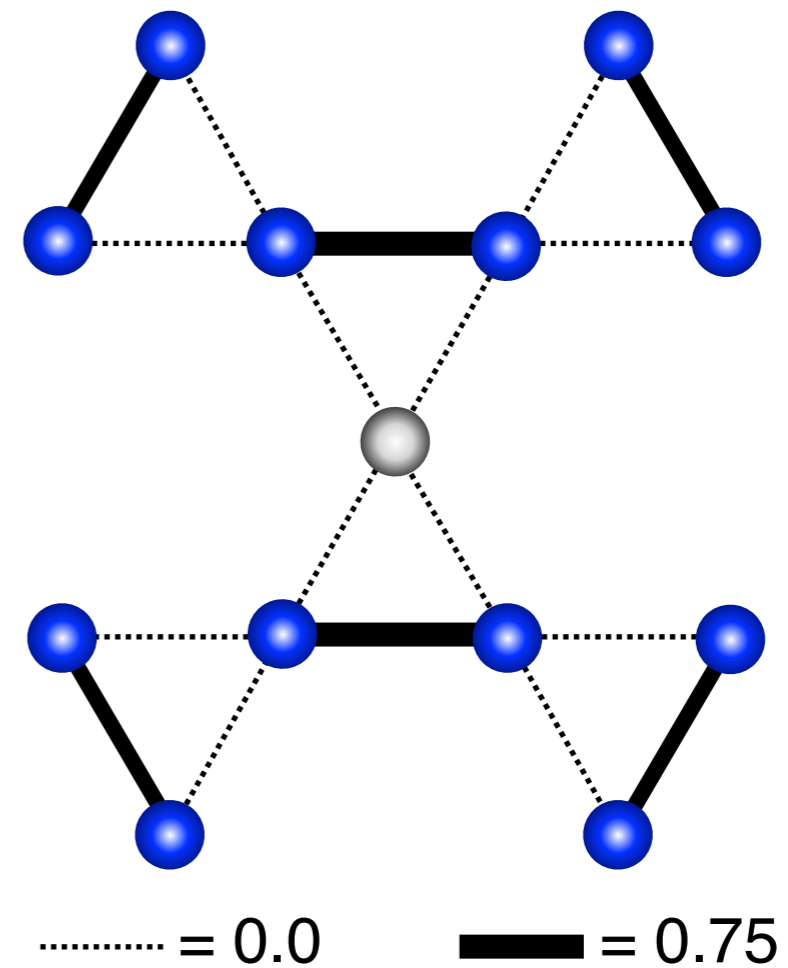
These look like encouraging signs of a QSL, but there are a few caveats...

0-6% Zn<sup>2+</sup>



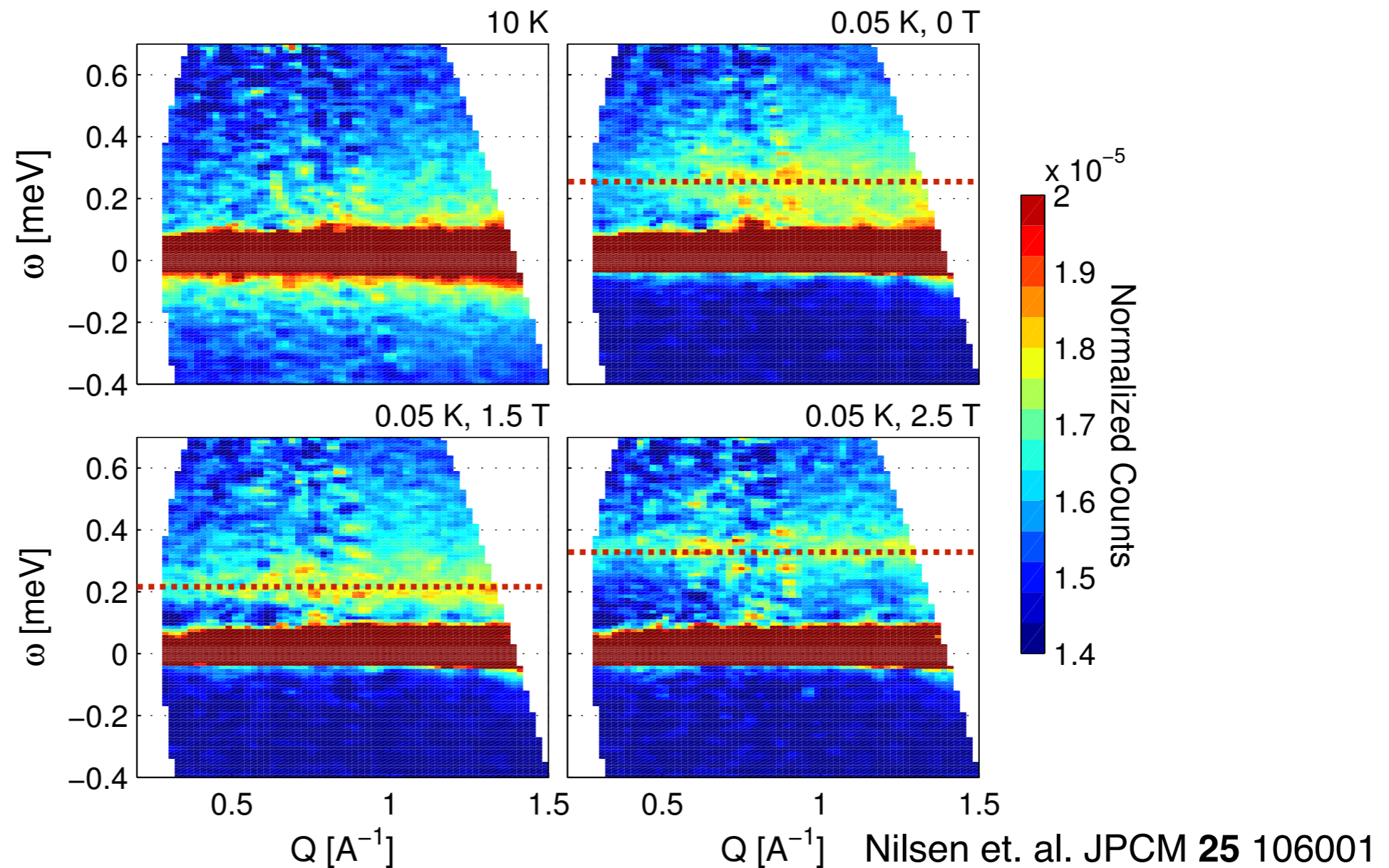
Freedman et. al. JACS **132** 16185  
de Vries et. al. PRL **100** 157205

~1 %, valence bond glass



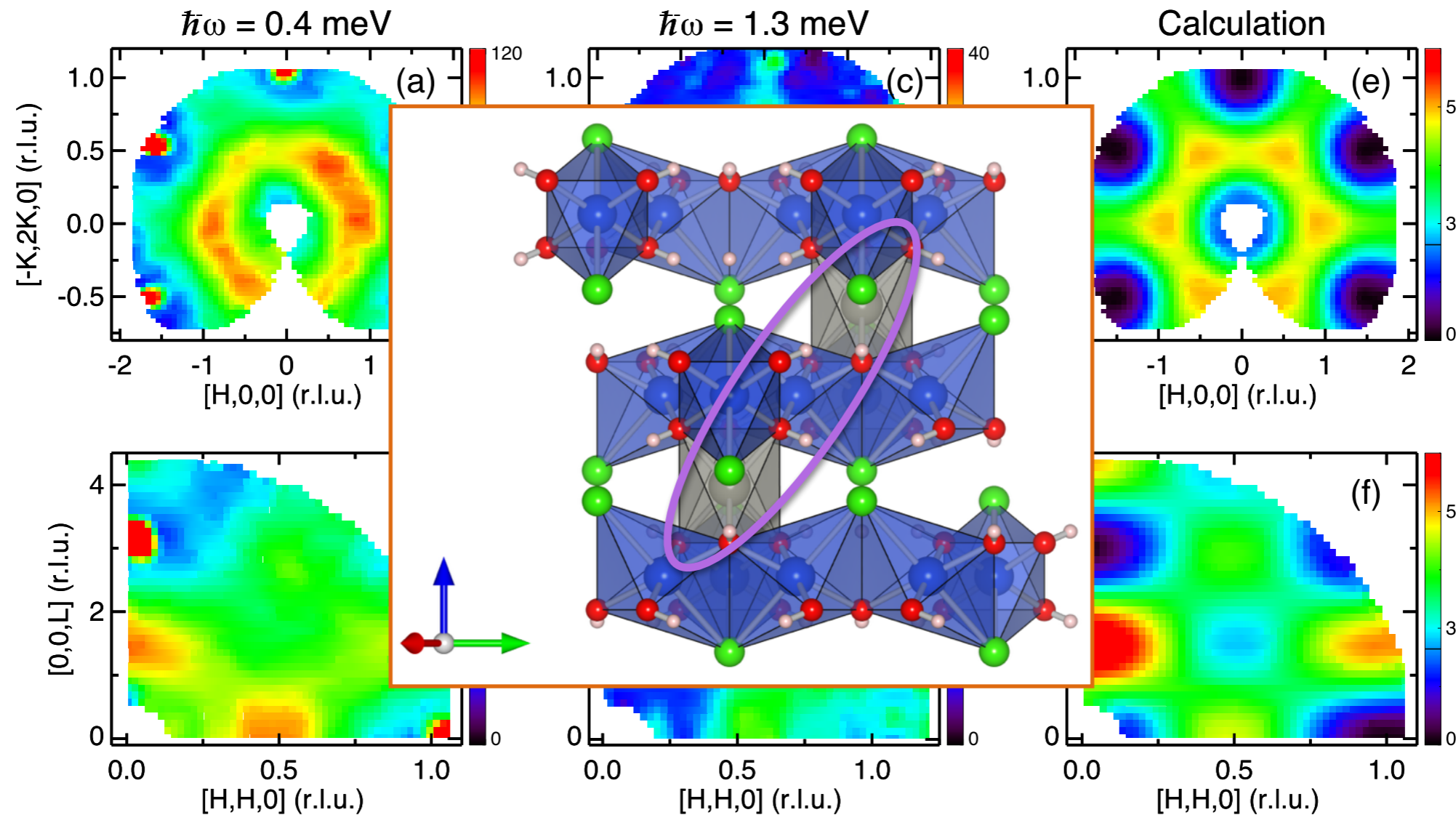
Singh, PRL **104** 177203

Low-energy  $S(Q, \omega)$  looks ungapped, although dominated by interplane defects



Defects weakly correlated, but can be decoupled with small magnetic field.

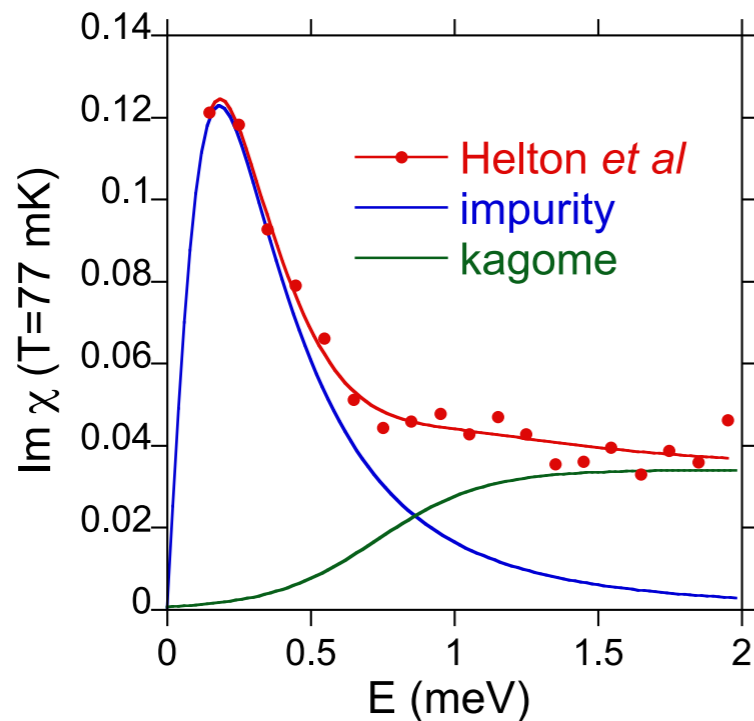
Single crystals add some detail to this picture.



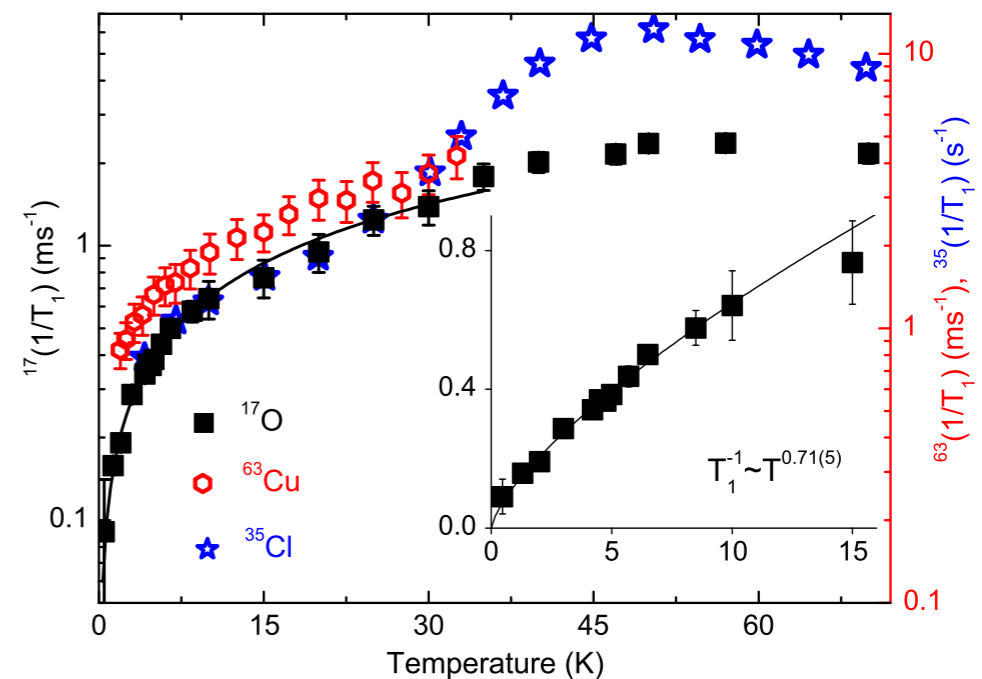
Han et. al. PRB **94** 160409

Antiferromagnetic coupling between interplane sites causes low-energy response.

1. Unclear whether spin gap is present or not - given large DM perturbation, expect gap to at least partially close.



Han et. al. PRB **94** 160409

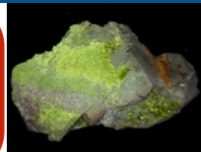



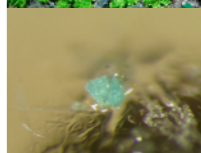



Olariu et. al. PRL **100** 087202

2. If gap is present, VBG or  $Z_2$  QSL? Entirely depends on how many defects are present on the kagome lattice

What other states can be realised in  
kagome minerals?

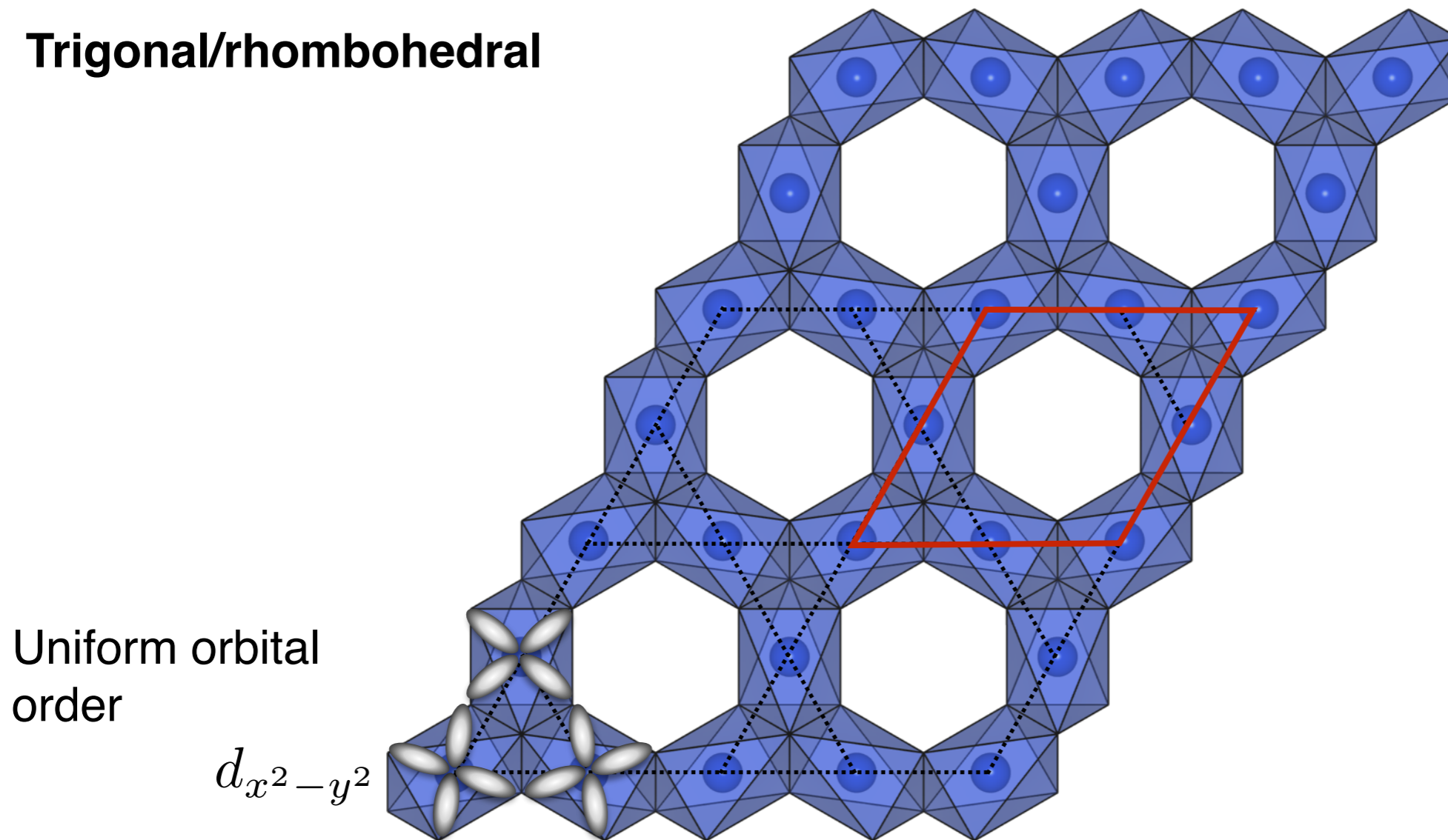
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Material	SG	$\theta$	Order	Reference(s)	
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Herbertsmithite $\beta\text{-Cu}_3\text{Zn}(\text{OH})_6\text{Cl}_2$	$R\bar{3}m$	-240 K	< 50 mK	M. P. Shores <i>et al.</i> JACS <b>127</b> , 13462 de Vries <i>et al.</i> PRL <b>103</b> , 237201 T. Han <i>et al.</i> Nature <b>492</b> , 406	
$\text{KCu}_3\text{As}_2\text{O}_7(\text{OD})_3$	$C2/m$	+13.4 K	7 K	Y. Okamoto <i>et al.</i> JPSJ <b>81</b> , 033707	
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Edwardsite $\text{Cu}_3\text{Cd}_2(\text{SO}_4)_2(\text{OH})_6 \cdot 4\text{H}_2\text{O}$	$P2_1/c$	- 50 K	4.3 K	H. Ishikawa <i>et al.</i> JPSJ <b>82</b> , 063710	
Kapellasite $\alpha\text{-Cu}_3\text{Zn}(\text{OH})_6\text{Cl}_2$	$R\bar{3}m$	0 K	< 50 mK	Colman <i>et al.</i> , Chem. Mater <b>20</b> 6897 Fåk <i>et al.</i> PRL, <b>109</b> 137208	
$\text{Cu}_3\text{Zn}(\text{OH})_6(\text{SO}_4)$	$P2_1/a$	- 79 K	< 50 mK	Y. Li <i>et al.</i> , cond-mat:1310.2795	



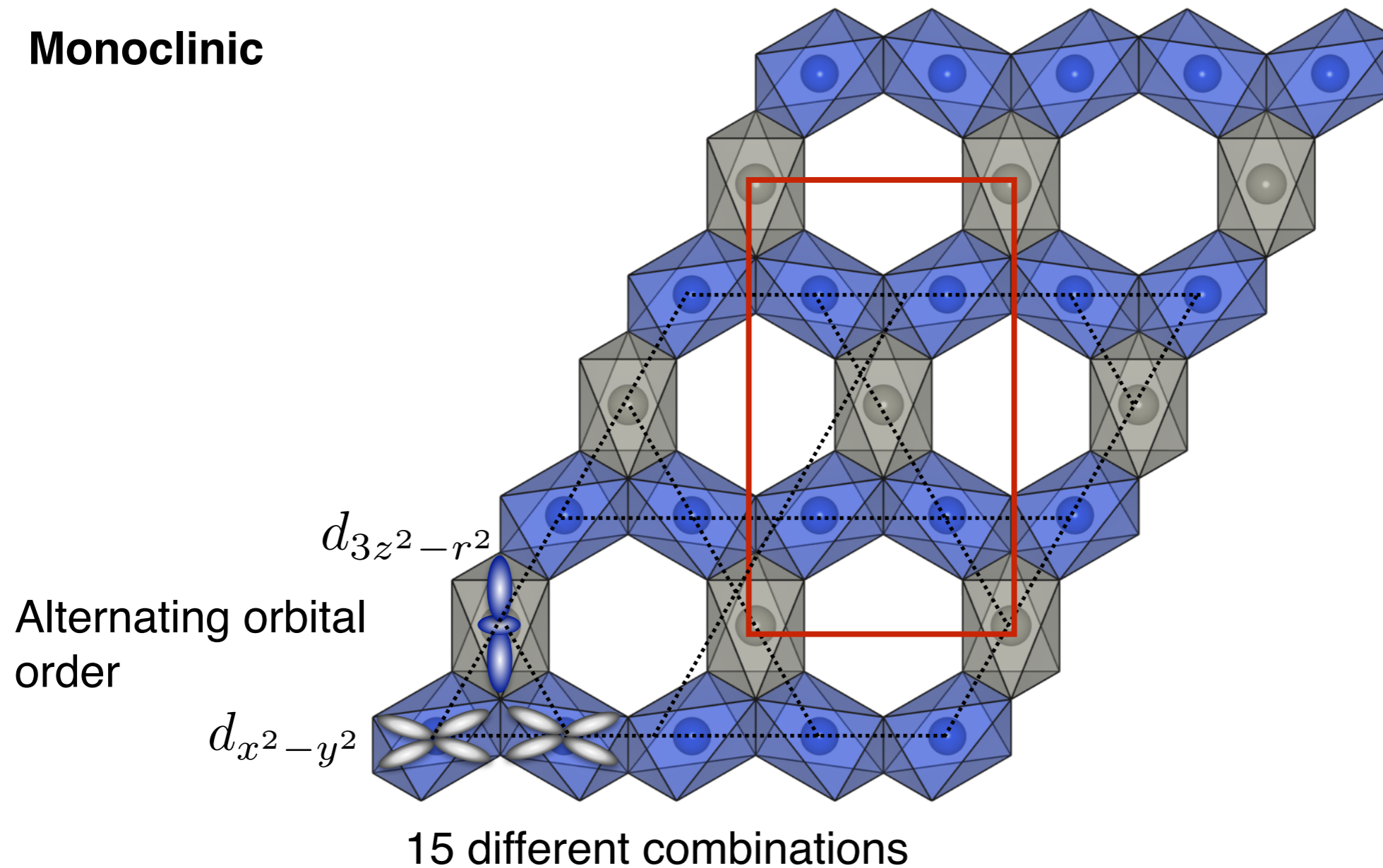
$\text{Cu}^{2+}$  is strongly Jahn-Teller active. Orbital can be inferred from local geometry

## Trigonal/rhombohedral



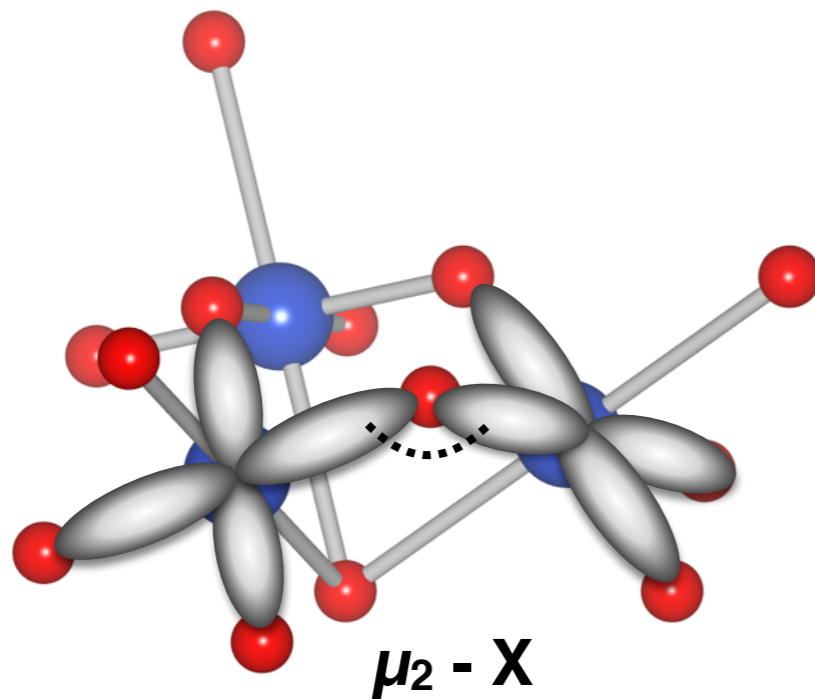
$\text{Cu}^{2+}$  is strongly Jahn-Teller active. Orbital can be inferred from local geometry

## Monoclinic



The orbital order not only leads to an anisotropy in the nearest neighbour bond distances, but also to fundamentally different exchange pathways:

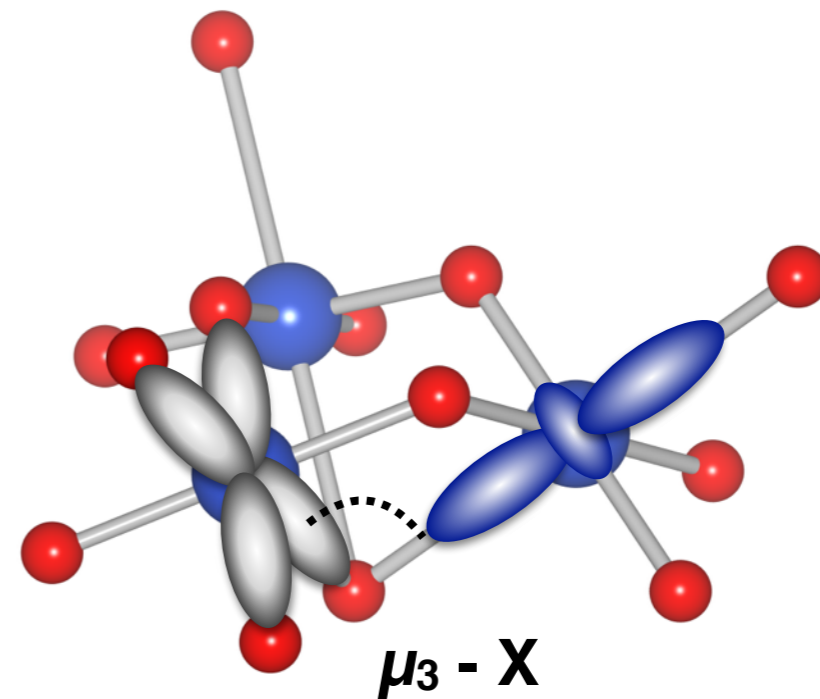
**e. g. uniform**



$\angle \text{Cu}^{2+} - X - \text{Cu}^{2+} \sim 120^\circ$

Antiferromagnetic

**e.g. alternating**

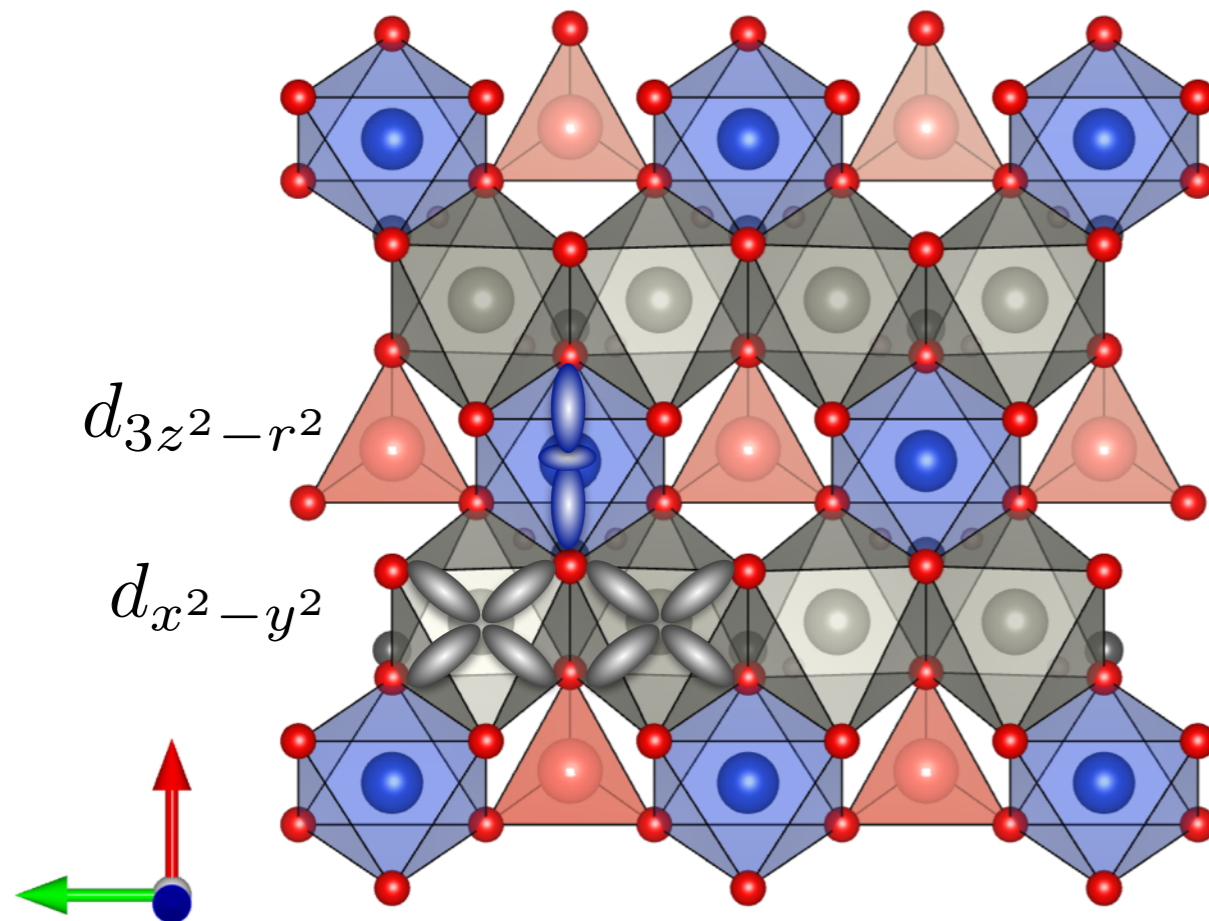


$\angle \text{Cu}^{2+} - X - \text{Cu}^{2+} \sim 90-105^\circ$

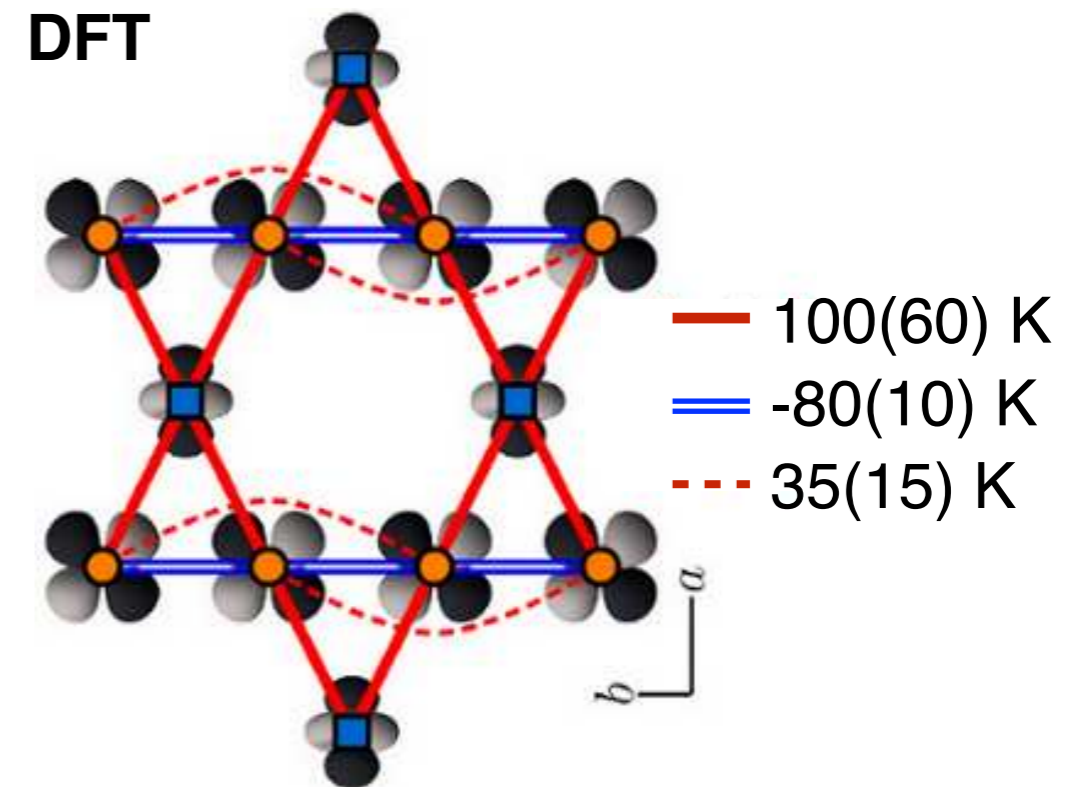
Can be ferromagnetic

Yoon et. al. Inorg. Chem **44** 8076

$C2/m$ ,  $\theta = -115$  K, orbital order



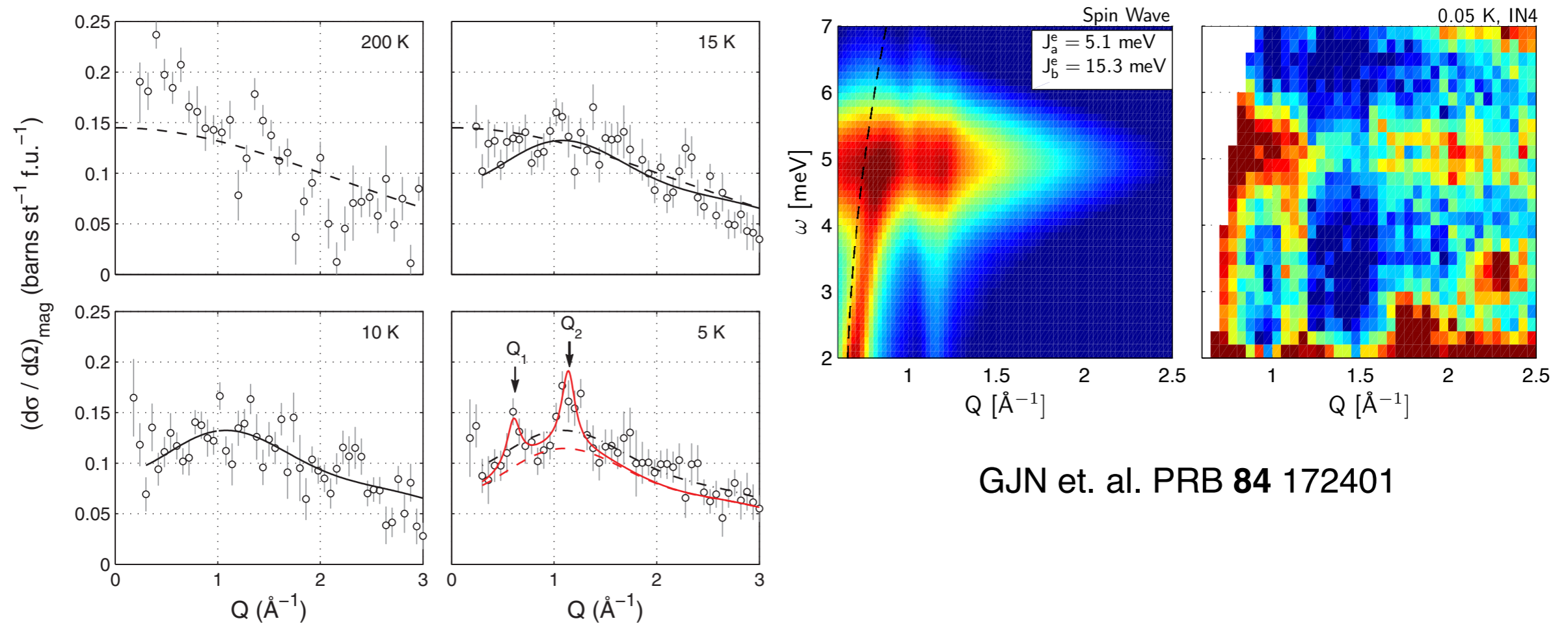
Hiroi et. al. JSPJ **70** 3377



Janson et. al. PRB **82** 134434

DFT calculations consistent with naive assumptions from coordination of  $\text{Cu}^{2+}$ :  
looks like volborthite can be described as coupled spin chains...

... however, despite being far away from the ideal kagome lattice antiferromagnet, volborthite shows no order down to low T

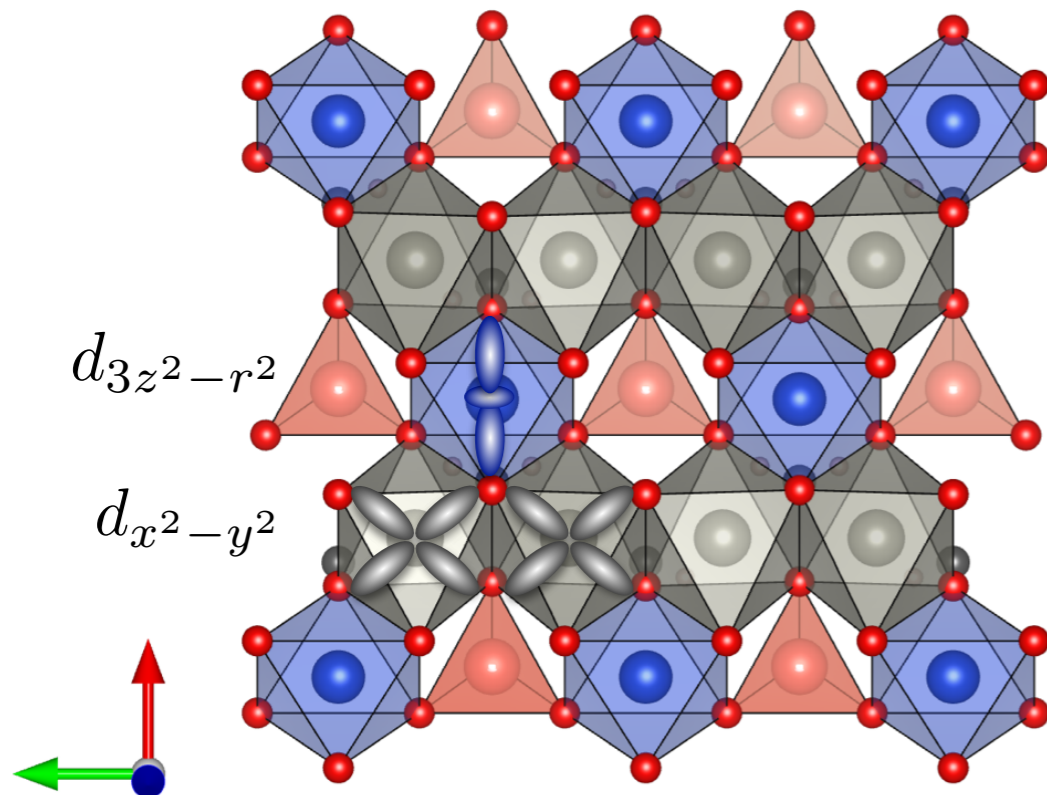


GJN et. al. PRB **84** 172401

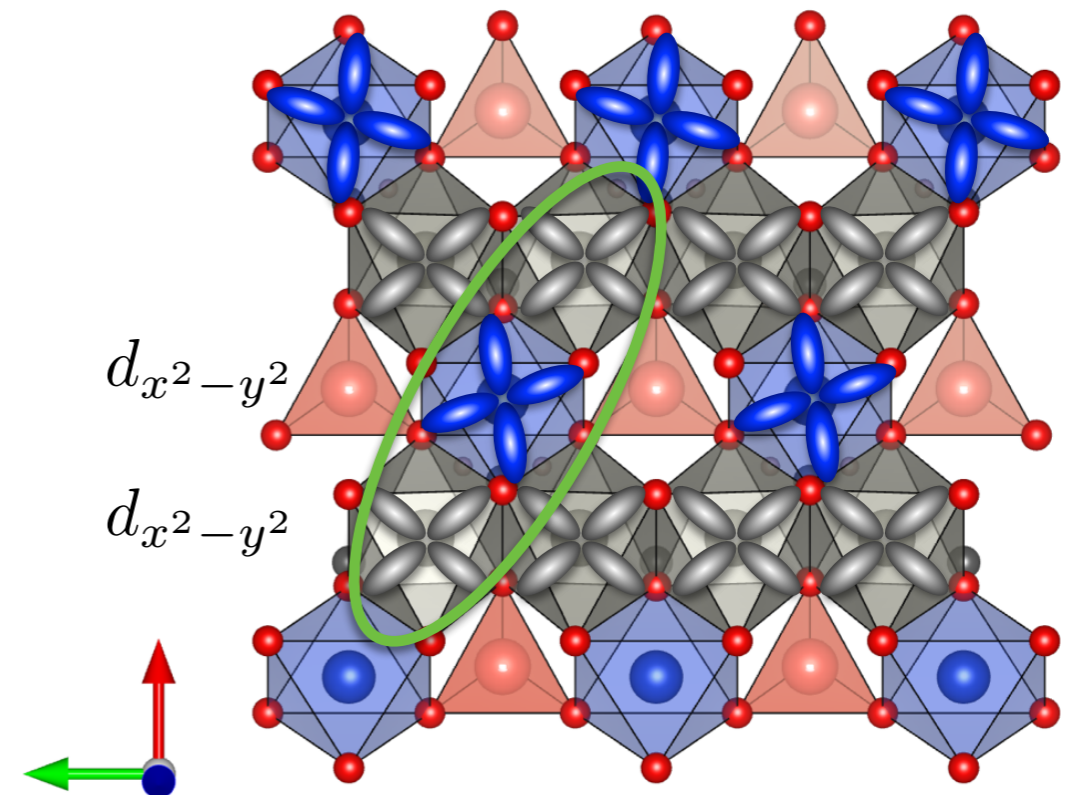
Only weak short-range order associated with incommensurate positions  $Q_1$  and  $Q_2$ , spin-wave-like spectrum at 50 mK. Order not captured by chain model.

A twist in the tale: high-quality powders and single crystals show an additional structural transition near room temperature:

**T > 310 K C2/m**



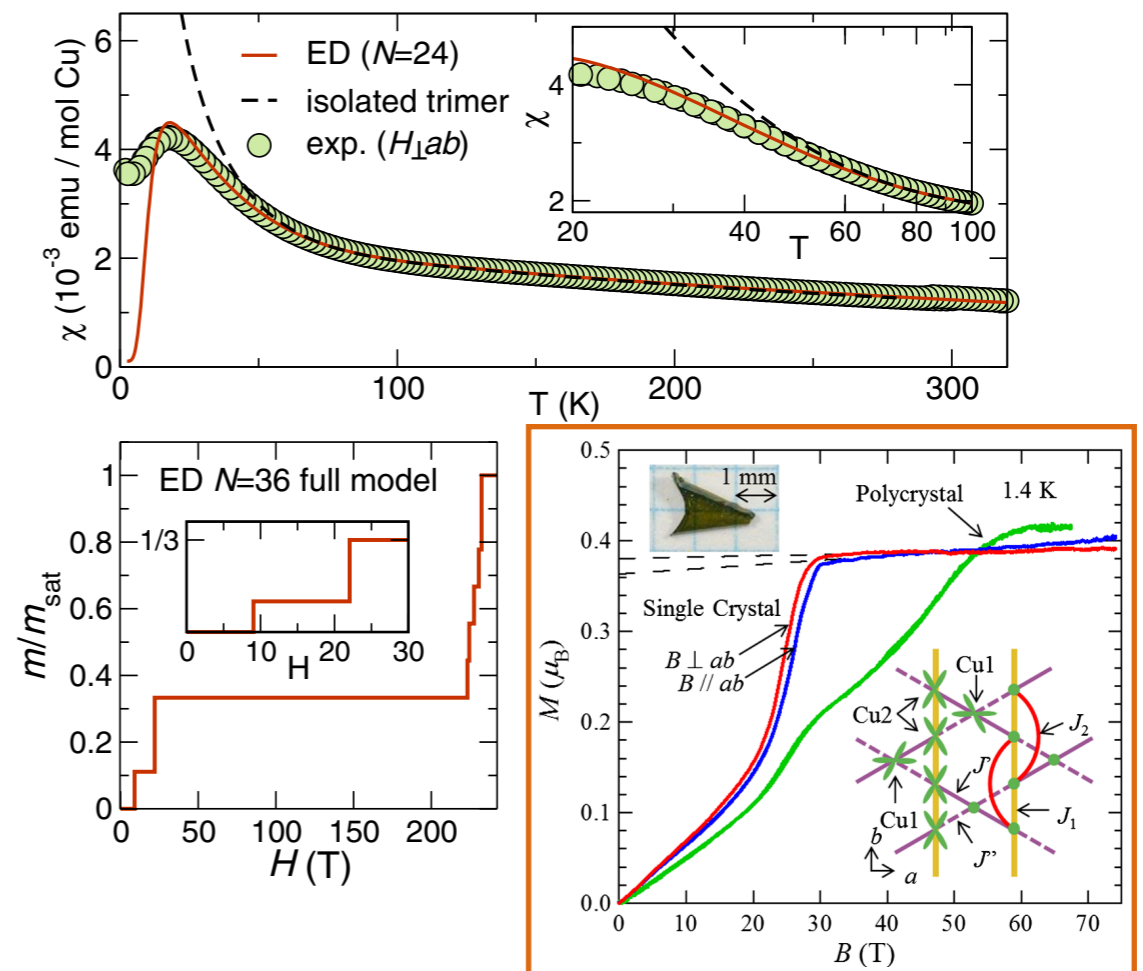
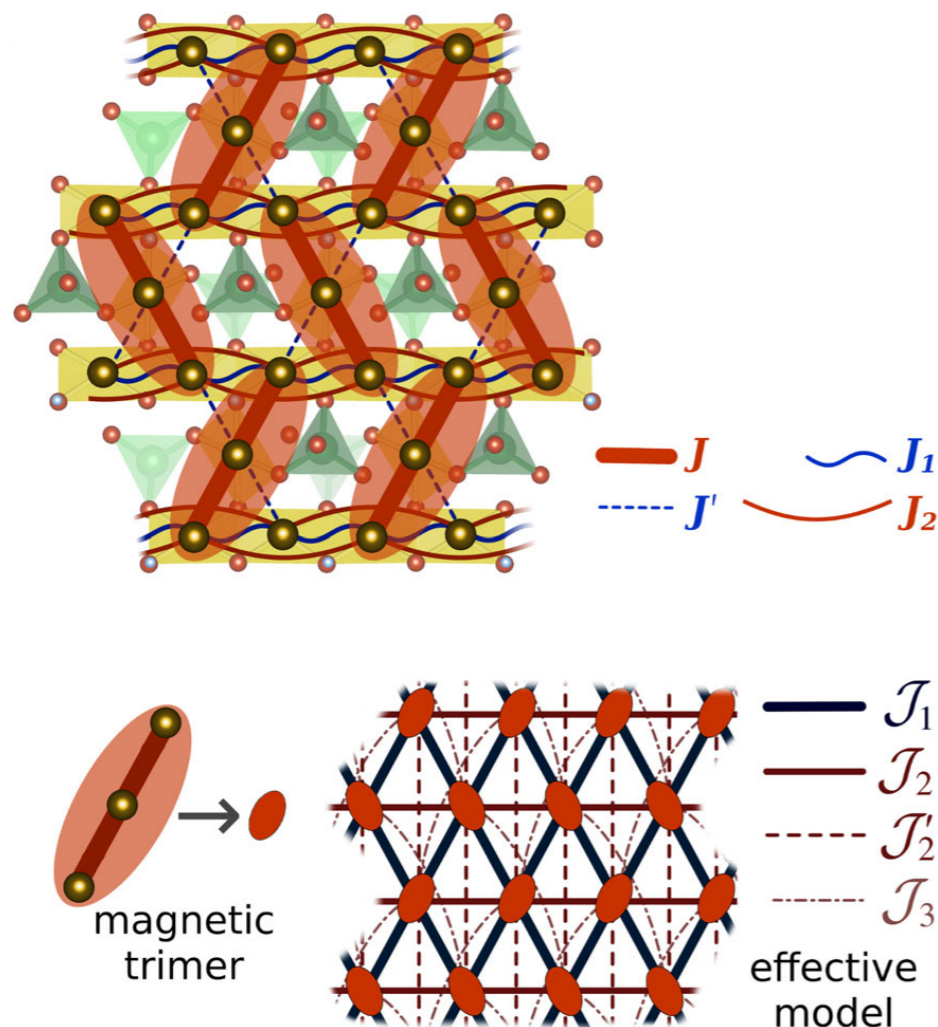
**T < 310 K C2/c**



This transition is an “orbital switching” — a very rare case for  $\text{Cu}^{2+}$  in an inorganic material, where such transitions usually occur at high T.

Yoshida, GJN et. al. Nature Comms. **3** 860

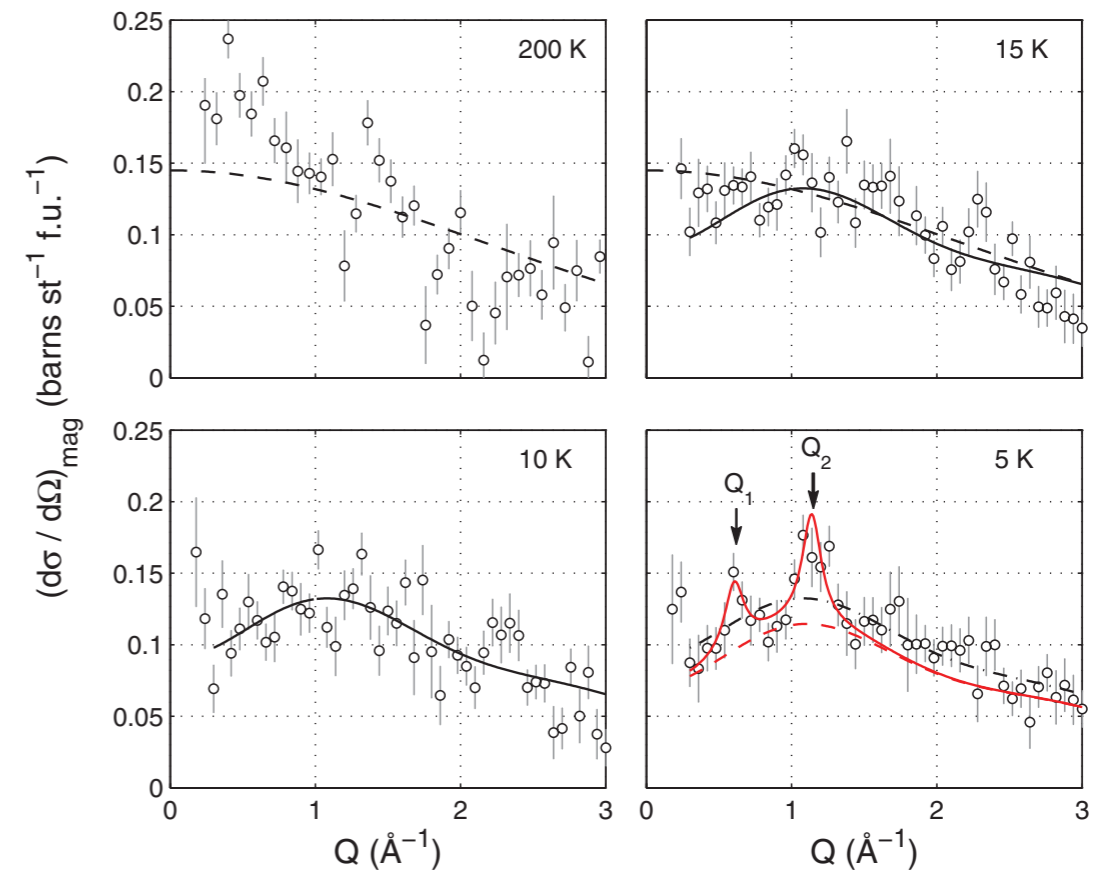
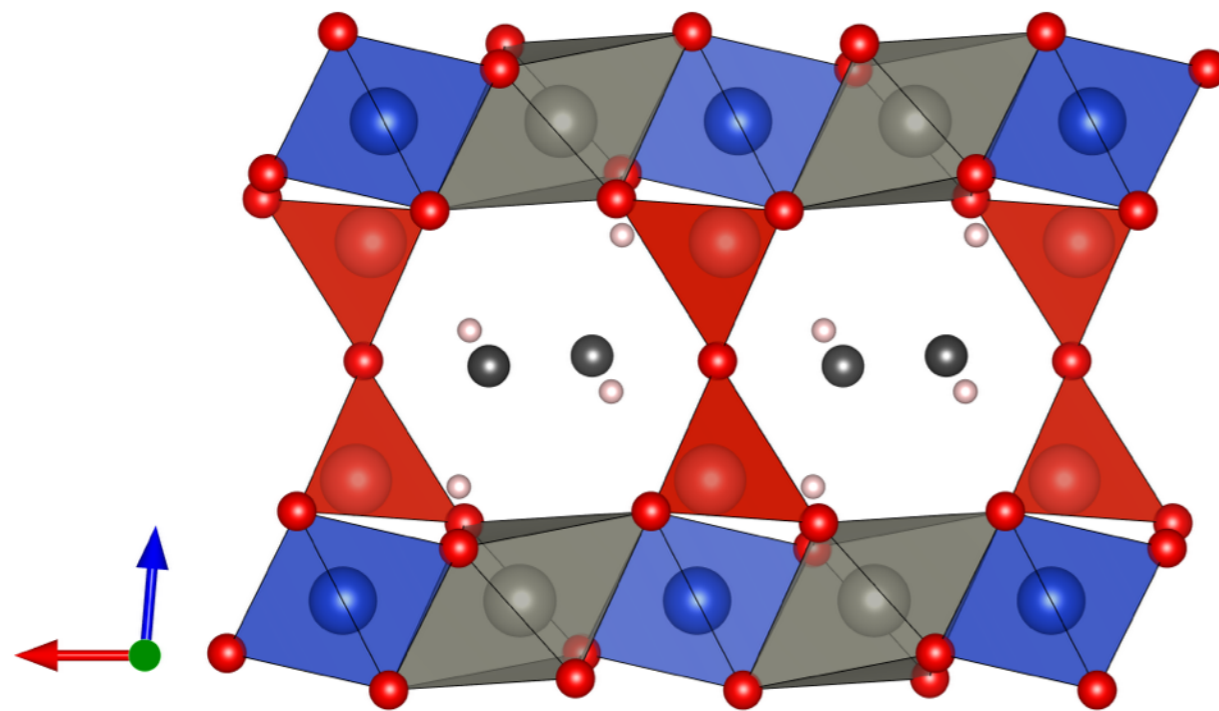
Interactions within trimers formed by orbital order dominate, resulting in effective triangular lattice system



Janson et. al. PRL **117**, 037206

Interesting bond nematic phase formed from condensation of two-magnon bound states at low  $H$ ...

Powders do not undergo “orbital switching” because of disorder (possibly related to disorder of interplane H<sub>2</sub>O)

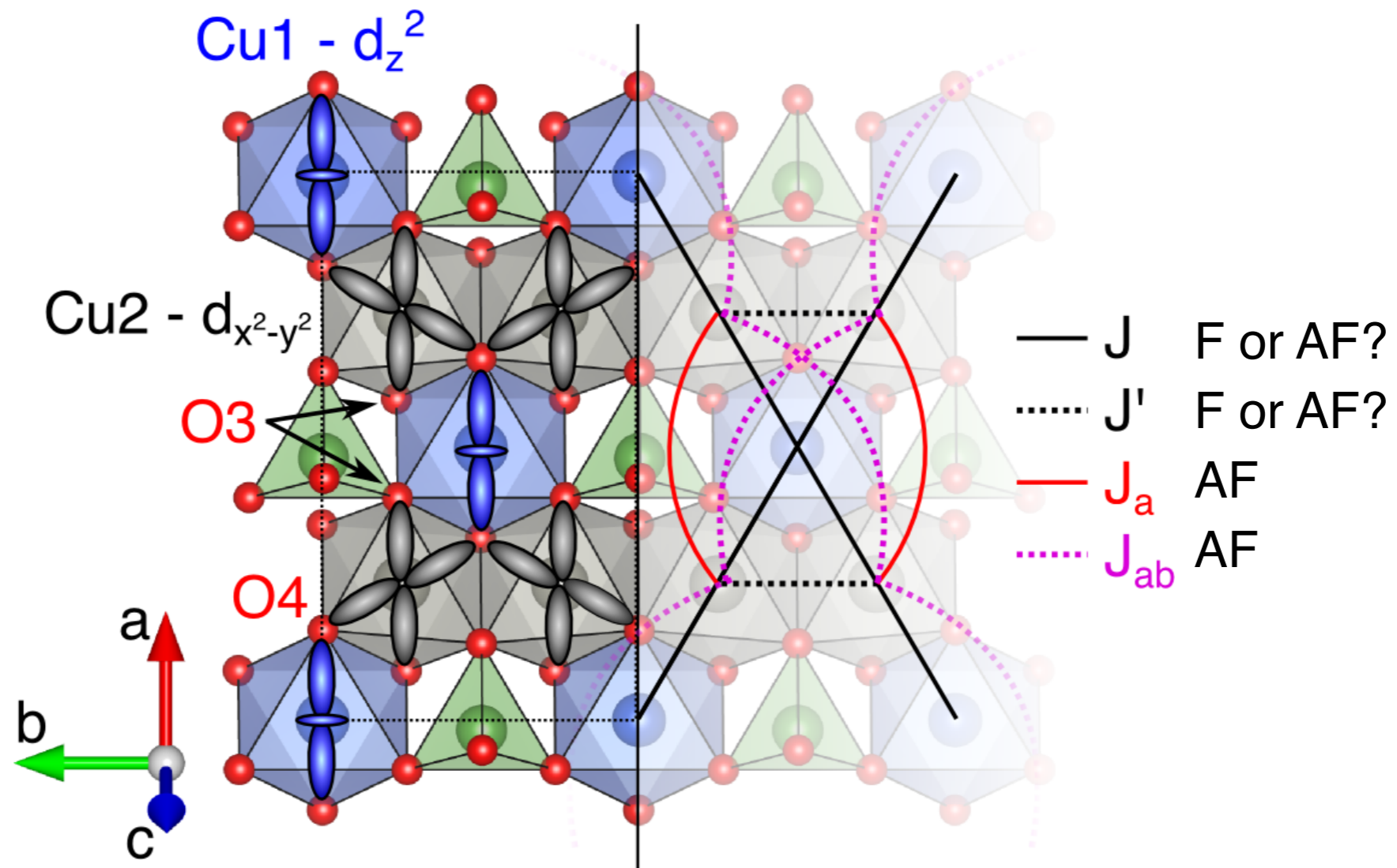


Diffuse scattering can thus be interpreted as small clusters (mostly dimers) + short-range magnetic order...



Similar orbital order to volborthite, but different orbital orientations:

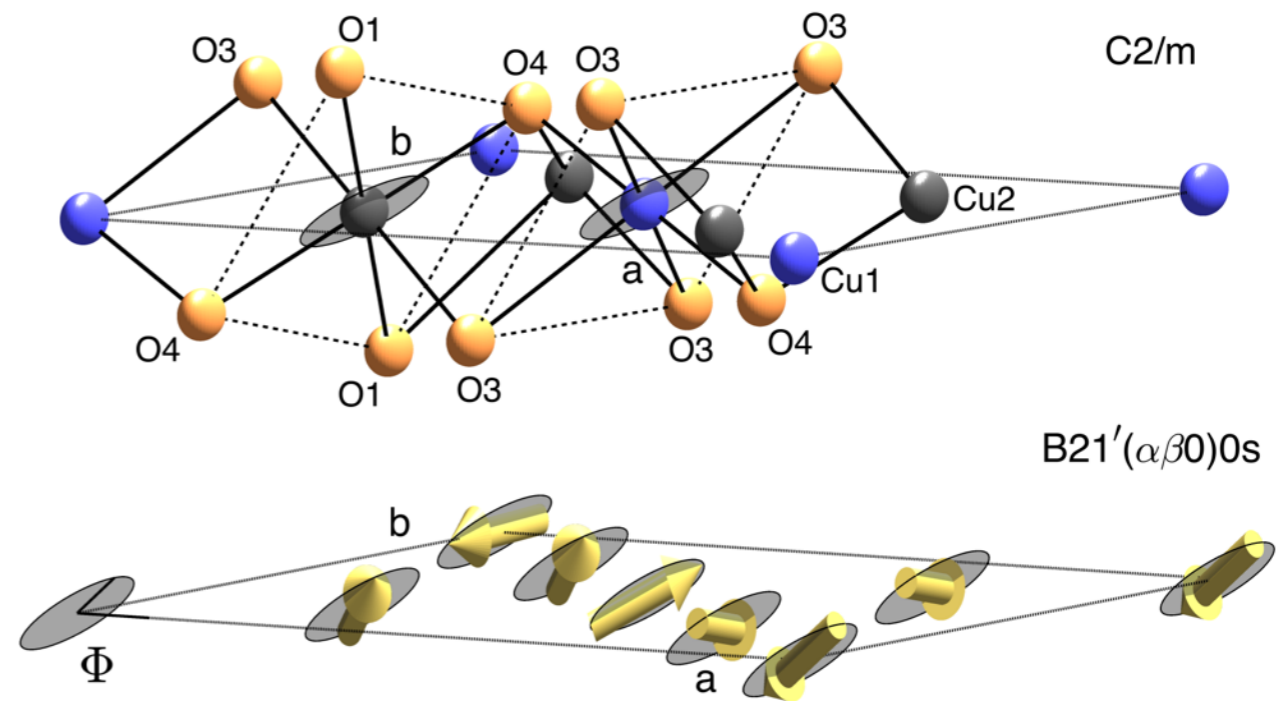
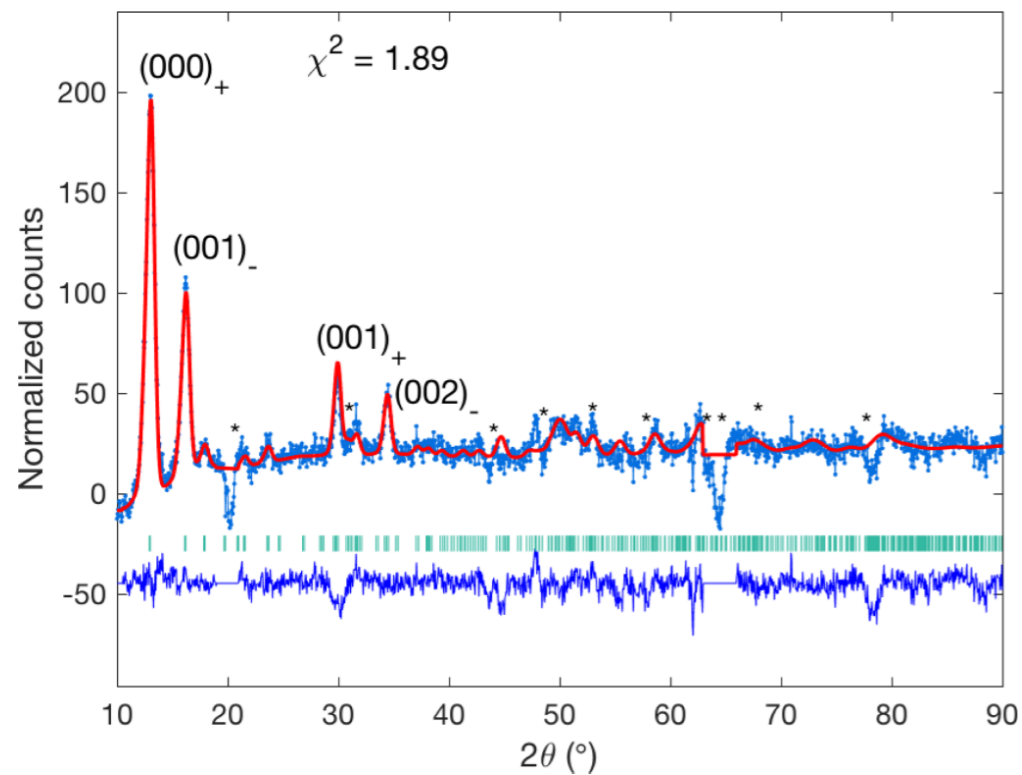
C2/m,  $\theta = +14$  K,  $T_N = 7.1$  K



Okamoto GJN et. al. JPSJ **81**, 033707

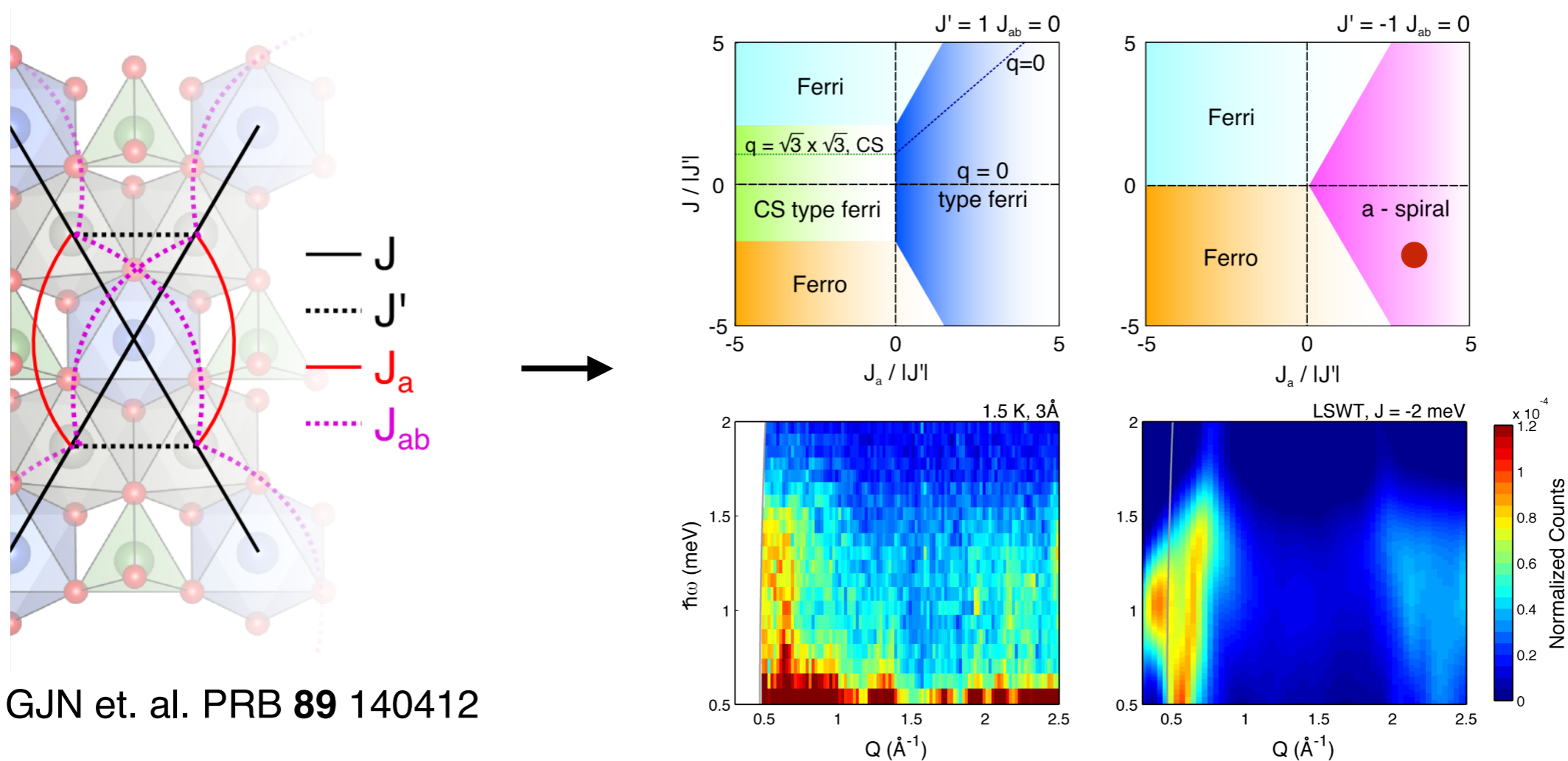
Unlike volborthite, magnetic order at  $T \sim \theta/2$  in a helical magnetic structure with propagation vector  $\mathbf{k} = (k_x \ 0 \ k_z)$  - there must be some frustration!

## D20, 1.8 K



GJN et. al. PRB **89** 140412 (R)

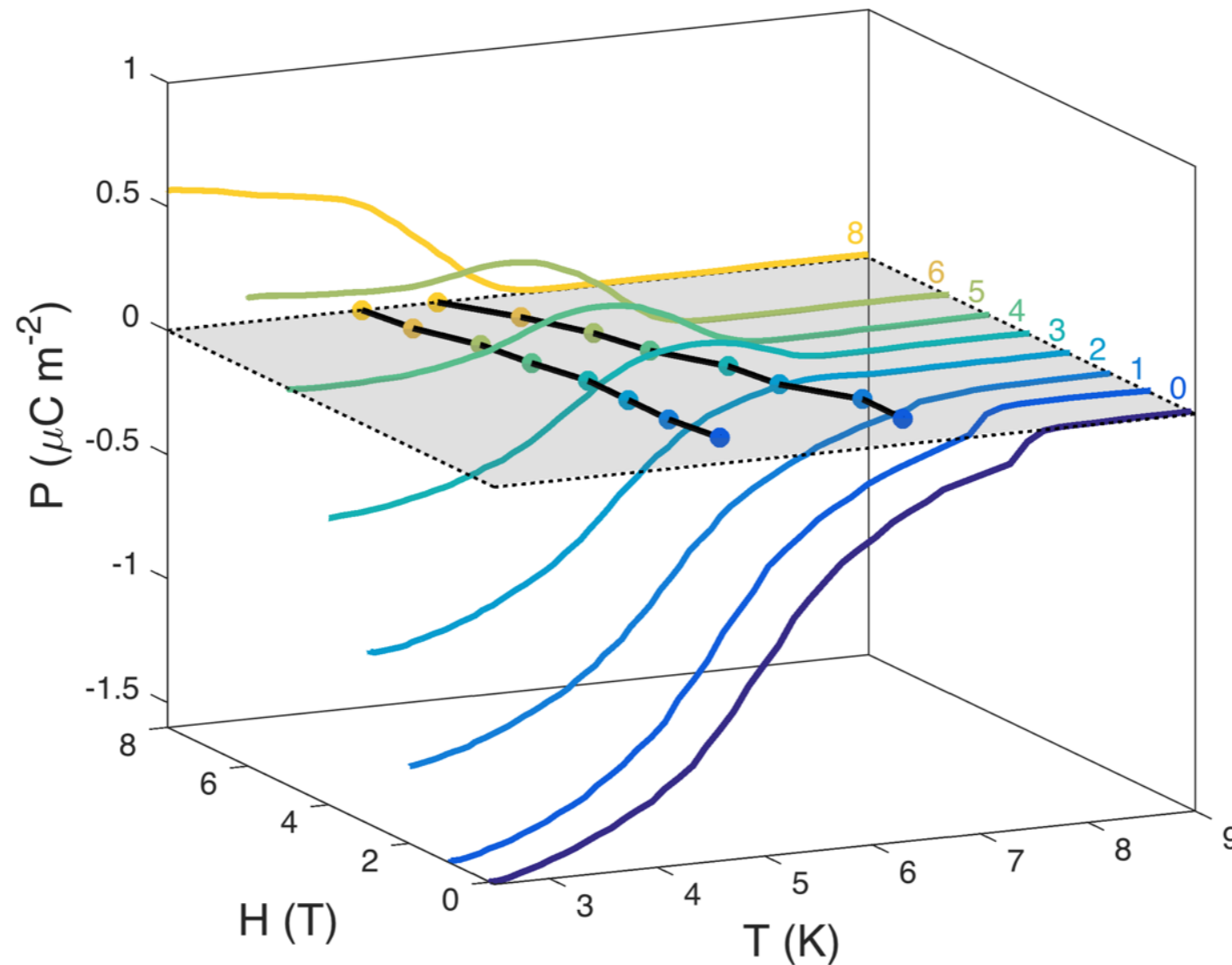
Unlike volborthite, magnetic order at  $T \sim \theta/2$  in a helical magnetic structure with propagation vector  $\mathbf{k} = (k_x \ 0 \ k_z)$  - there must be some frustration!



GJN et. al. PRB **89** 140412

Both nearest neighbour interactions ferromagnetic! Frustration from antiferromagnetic further neighbour couplings.

The polar point group ( $21'$ ) of the magnetic structure permits a ferroelectric polarization:



GJN et. al. PRB **95**, 214415

- The kagome lattice antiferromagnet is the premier magnetic model for new quantum many-body states
- Most kagome lattice materials studied so far have been  $\text{Cu}^{2+}$  minerals, which allow for several perturbations beyond the nearest neighbour coupling
- Among these:
  - Herbertsmithite is very close to the ideal kagome lattice antiferromagnet
  - Volborthite shows orbital reorientation and trimerization
  - $\text{KCu}_3\text{As}_2\text{O}_7(\text{OD})_3$  is far away from QSL, but still frustrated and multiferroic
- These large differences in behaviour can be traced back to the orbital occupation and consequent superexchange pathways
- Despite the difficulty realising a QSL in kagome minerals, new frustrated behaviours often result because of the topology of the kagome lattice
- Future work: other candidates, charge doping...



We're not at the top yet, but at least now we can enjoy the view



**Yoshihiko Okamoto**  
**Zenji Hiroi**  
Hajime Ishikawa  
Ryutaro Okuma  
Hiroyuki Yoshida...



**Mark de Vries**  
**Fiona Coomer**  
**Andrew Harrison (DLS)**



**Pascale Deen (ESS)**  
**Ross Stewart (ISIS)**  
Laurent Chapon  
Thomas Hansen  
Hannu Mutka



**Virginie Simonet**  
**Claire Colin**



**Henrik Rønnow**

... and thank you for your attention!